

External vulnerability and optimal monetary policy in Nigeria

Sunday Oladunni^{1,2}

In this study, we assess the external vulnerability of the Nigerian economy by documenting three alternatives (zero, partial and full) oil price pass-through to inflation within a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) framework. The results show that under a zero-oil price pass-through, the choice of inflation measure is immaterial, as macroeconomic responses to the shock are comparable under alternative Taylor rule specifications. The shock precipitates stagflation, transmitted via the income and exchange rate channels; and introduces an extra layer of vulnerability associated with higher external risk premium. Both core and oil inflation targeting monetary rules maximize welfare under a zero-oil price passthrough, while oil inflation targeting is shown to be welfare superior under partial or full oil price pass-through. Credibility consideration renders core inflation targeting the feasible optimal path for the central bank.

Keywords: External vulnerability, New Keynesian DSGE, Monetary Policy, Oil Exporting Small Open Economies

JEL Classification: Q35, F47, E52, F41

DOI: 10.33429/Cjas.01024.2/7

1. Introduction

This paper analyses principal source(s) of external vulnerability and their effects on the Nigerian economy through the lens of a New Keynesian dynamic stochastic general equilibrium (DSGE) model. The model reflects stylized features of a typical small open, emerging, and crude oil-exporting economy which in turn imports refined oil for consumption and production.

External sector vulnerability has been at the core of most emerging and developing economies business cycle fluctuations for many decades. However, the state of the domestic economy determines to a large extent, the size of the impact and amplification of foreign shocks. Inherent structural weaknesses, inefficiencies, and macroeco-

¹Email address: sunday.oladunni@alumni.ucalgary.ca

²The author is staff of the Alberta Ministry of Treasury Board and Finance, Canada. Views expressed in this paper are those of the author and do not represent the position of the institution where he is employed

conomic imbalances in the domestic economy tend to exacerbate the impact of adverse external shocks, thereby undermining domestic resilience to such shocks. The situation in which many oil-dependent small open economies rely predominantly on crude oil export for foreign exchange and government revenue is a pointer to the delicate economic structures in these countries. A large proportion of the expected positive inter-sectoral spillovers, forward and backward linkages from oil production do not materialize. This is because, oil sectors in these economies operate as enclave sectors, attracting vast amount of production inputs from abroad and producing almost entirely for export. In addition, unanticipated and prolonged adverse oil price shocks tend to create external vulnerabilities which often snowball into sharp output declines, high inflation, fiscal insolvency, currency crises, and welfare losses. Consequently, tough fiscal choices and appropriate monetary policy responses are required to ameliorate the welfare effects of oil-induced vulnerabilities among oil exporting small open economies.

The literature on micro-founded models with oil price shocks, macroeconomic dynamics and monetary policy in net oil exporting small open economies is rather sparse. Medina & Soto (2005), in an estimated New Keynesian small open economy dynamic stochastic general equilibrium (DSGE) model for Chile, incorporated oil in the household consumption bundle and in domestic firm's production function and found that a positive oil price shock resulted to higher inflation and lower output. An oil-induced output decline is linked to the endogenous tightening of monetary policy. They also reported that a wage stabilizing monetary policy rule is welfare superior to those targeting core and headline inflation measures, albeit at significant output cost. Poghosyan & Beidas-Strom (2011) estimated a DSGE model for Jordan which feature price and wage rigidities; imported oil as consumption good and production input; and found that oil price shock caused a huge negative income effect, exchange rate depreciation and current account improvements. They found also that a pegged exchange rate regime delivers a comparatively low risk premium, with consequential amplifications in consumption, output, and inflation volatilities. More recently, Hollander *et al.* (2019) estimated a New Keynesian DSGE model for South Africa, with oil as a consumption good and a production factor; and reports that real oil price shock's effect on output and consumption is significant and persistent. Thus,

they concluded that oil price shock is a fundamental driver of inflation, output, and interest rates; given that it generated a trade-off between inflation and output stabilization. Their results also suggest that endogenous monetary tightening in response to oil price shock undermined economic recovery in South Africa¹.

This paper, however, sets up a New Keynesian small open economy DSGE model which embeds some fundamental features of a net oil exporting developing economy. The domestic oil sector produces crude oil exclusively for export; while households and non-oil firm's consumption basket and production function, respectively, feature refined oil imported from the rest of the world. We assume nominal price rigidity in the domestic goods sector (Gali & Monacelli, 2005), a competitive labour market (Hove *et al.*, 2015), the operation of the law of one price gap (Monacelli, 2005 and Burstein & Gopinath, 2014), a perfectly competitive, non-exogenous² enclave oil producing sector, imperfect international risk-sharing, induced endogenously by a debt-sensitive external risk premium and an exogenous oil price shock; oil subsidy (Bouakez *et al.*, 2008 and Allegret & Benkhodja, 2015), and a monetary policy setting that features four alternative policy rules popular in most emerging markets and developing economies.

The paper contributes to the literature in three ways. First, we capture fundamental features of commodity exporters that re-import the commodity they export in another form after foreign value addition; thereby leading to job export, low competitiveness, tax burden of foreign origin, in addition to the vulnerability associated with oil earnings volatility³, imported inflation, and foreign exchange pressures. Second, we establish a direct connection between crude and refined oil prices. This is not the case with models by Medina & Soto (2005), Poghosyan & Beidas-Strom (2011) and Hollander *et al.* (2019), which were oriented towards oil importing small open economies that exports non-oil commodity endowments. Consequently, they did not explore the interaction⁴ between the two price dynamics. In addition, given that the

¹All three models see the world from the angle of an oil importer

²Crude oil output is modeled explicitly, as opposed to the popular trend of simply assuming it to be an exogenous process.

³Due to commodity price fluctuations

⁴The interaction between exported crude and imported petroleum prices is key to underlining external vulnerability in our model

economies of Chile, Jordan and South Africa modeled, respectively, in the papers are fundamentally oil importing, their model features can not approximate the structural realities in SOEs that both export and import oil. Third, we highlight the seeming structural chasm between domestic oil and non-oil sectors in some net oil exporting countries; by assuming that crude oil is exported wholly to the rest of the world and that oil sector attracts capital from the rest of the world in form of foreign direct investments (FDI). Our characterization shows the near-zero direct interaction between oil and non-oil sectors among many developing oil exporters and captures the low levels of industrialization in these countries.

Models developed for advanced oil exporting countries such as Dib (2008) for Canada; and Ferrero & Seneca (2015) and Bergholt *et al.* (2017) for Norway cannot be situated in the context of the net oil exporting developing and emerging economies. This is because they are enriched with strong domestic inter-sectoral industrial interactions which often generate significant positive spillovers between the oil sector and the mainland economy. They generally tend to embed fiscal rules designed to promote national savings and de-link the domestic economy from the direct effects of oil price volatility. These features have not yet taken roots in many oil exporting developing countries. The closest strand of literature to our model includes Allegret & Benkhodja (2015) for Algeria, Algozhina (2016) for Kazakhstan and Iklaga (2017) for Nigeria; nonetheless, they do not account for our contributions. For instance, in the case of Nigeria, Iklaga (2017) assumed a wholly exogenous oil sector, in which both oil output and price evolve exogenously. Whereas in our model, crude oil production is endogenous, allowing the oil firm to utilize domestic labour and imported capital in its production function. Only the oil price is exogenous. Furthermore, to the best of our knowledge, this is the first attempt to highlight the relationship between crude and refined oil prices.

We simulate a negative oil price shock and analyze the consequent macroeconomic responses under alternative monetary policy specifications and oil price pass-throughs. Using optimized simple rules, we test for the welfare implications of implementing the alternative policy rules under three oil price pass-throughs or subsidy scenarios. Following the shock, the economy experiences stagflation. Stagflation manifests

via the income and exchange rate channels. External debt issued by households to cushion the combined effects the fall in income, higher lump-sum tax, higher unemployment, volatile wage, and inflation elevated the economy's risk premium. In a full subsidy environment where oil price pass-through is nil, the choice of target variable in the Taylor rule appears not to matter, as macroeconomic responses to the shock under all monetary policy specifications exhibit extreme similarity; a development which suggests that oil subsidy impairs monetary policy transmission mechanism. Given free labour mobility, the adverse shock to the oil sector encouraged movement of workers from the sector to the non-oil sector, thereby boosting non-oil sector productivity and output. The central bank responds to inflation and exchange rate pressures by raising the interest rate.

Macroeconomic fluctuations under partial and zero subsidy regimes follow similar directional patterns as under the full subsidy regime, but response speed and magnitudes are more pronounced under the former. Although, the monetary policy rule with oil inflation target is associated with less sharp impact response of oil consumption, aggregate consumption, and aggregate output, it is characterized by slightly higher volatility over time, compared to other monetary policy rules. Monetary policy response was least aggressive on impact under oil inflation targeting rule but later became aggressive as the initial fall in oil inflation reversed. Tight external borrowing condition added an extra source of external vulnerability to the negative oil shock. The optimized simple rules policy exercises show that either core or oil inflation targeting maximizes welfare in a full subsidy scenario, while targeting oil inflation is shown to be welfare superior assuming a partial or zero subsidy scenario. This outcome presents a challenge in a rule-based interest rate setting environment, as the policy maker may jeopardize its credibility as it responds to movement in oil price, an exogenous and highly volatile variable. Our results suggest that oil subsidy can play a role in moderating adverse oil shock-induced business cycle fluctuations and can be welfare maximizing, albeit, assuming the efficiency of the oil subsidy administration.

The rest of the paper is organized as follows. Following the introduction is Section 2, which describes model environment and equations underpinning the behaviour of

agents and the resulting equilibrium conditions. Section 3 presents model calibration, solution, and simulation, while in Section 4, we analyze simulation results. In Section 5, we evaluate welfare under the optimized simple rules and Section 6 concludes the paper.

2. Model

2.1 Model Environment

The model is built for an oil-endowed small open economy with core structural characteristics of an emerging or a developing economy with resource endowment (see Figure 1). There is a representative household whose consumption bundle includes imported refined oil, domestic and foreign goods. The household accesses international capital market for consumption smoothing purposes. However, an endogenous external debt-sensitive risk premium term introduces a financial friction which does not allow for perfect international risk sharing.

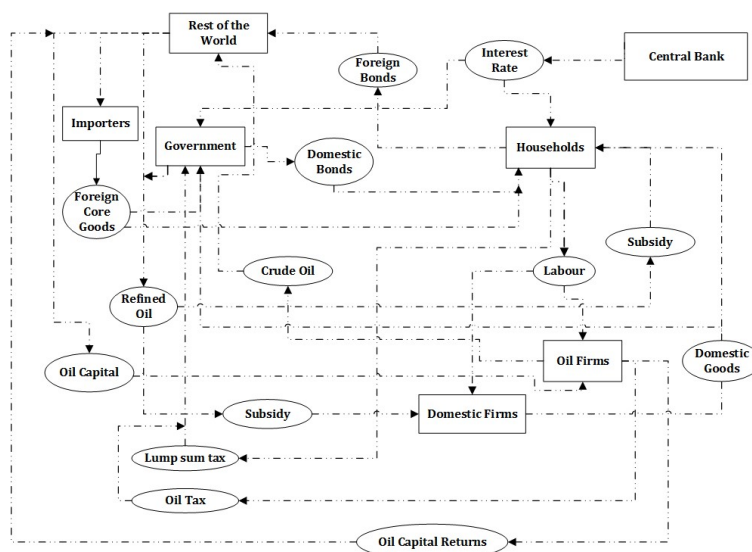


Figure 1: Overview of the Model

On the firm side, two categories of representative firms exist; the first is a monopolistically competitive firm that combines labour and refined oil to produce core goods for domestic consumption and export; and the second operates as a perfectly competitive firm, utilizing a production function that includes labour and foreign oil capital

to produce crude oil exclusively for export. There is an importer with a pricing mechanism based on Calvo (1983) just like the domestic monopolistically competitive domestic firm and import price is subject to the law of one price gap in the spirit of Monacelli (2005), which reflects exchange rate incomplete pass-through.

The labour market is assumed to be competitive, allowing perfect cross-sector mobility of workers. The domestic assets market functions to ensure a zero net supply of domestic bonds. The government levy lump-sum tax on households and oil tax on oil producing firm’s net revenue. The post-tax balance of oil firm’s net revenue constitutes returns on foreign capital and is paid to foreign direct investors. In addition, the government consumes domestic and foreign goods, and can either spend or save on oil subsidy, depending on the direction of foreign oil price movements. It also provides sovereign guarantees for households’ external debt obligations.

The central bank cares about agents’ welfare and sets the interest rate according to the Taylor rule specification(s) that shows the objective to stabilise output and inflation or exchange rate. The policy maker must choose the inflation measure to target as an instrument variable in the Taylor rule. For the model economy, the variants we consider include headline inflation, core inflation, and oil inflation. Either of the three is to be combined with either output or the real exchange rate to anchor inflation expectations, promote macroeconomic stability and maximize welfare in the economy. Our optimal monetary policy exercise compares outcomes of optimized simple rules given a negative oil price shock under alternative monetary policy specifications and oil subsidy regimes. The rest of the world is assumed to evolve exogenously.

2.2 Households

The economy is inhabited by a continuum of infinitely lived households indexed by $i \in [0, 1]$. Following the representative agent modeling approach referenced in Gali (2018), we model a typical Ricardian representative household, who has access to both domestic and foreign capital markets. However, risk sharing is imperfect owing to an external risk premium. Household total consumption bundle (C_t) is a composite of core (non-oil) goods and refined oil; represented as follows:

$$C_t = \left[(1 - \Psi_{ro})^{\frac{1}{\nu}} (C_t^c)^{\frac{\nu-1}{\nu}} + \Psi_{ro}^{\frac{1}{\nu}} (C_t^{ro})^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}} \tag{1}$$

Where C_t^c is core consumption, C_t^{ro} is (refined) oil consumption, Ψ_{ro} and $1 - \Psi_{ro}$ are shares of refined oil and core goods, respectively in the composite consumption basket, while ν represents the elasticity of substitution between core and oil consumption. Core consumption, following a constant elasticity of substitution (CES) aggregator, is composed of domestically produced and imported goods. It is given as follows:

$$C_t^c = \left[1 - \Psi_f \frac{1}{\varpi} (C_t^h)^{\frac{\varpi-1}{\varpi}} + \Psi_f \frac{1}{\varpi} (C_t^f)^{\frac{\varpi-1}{\varpi}} \right]^{\frac{\varpi}{\varpi-1}} \quad (2)$$

Where C_t^h and C_t^f are the bundles of domestically produced and imported core goods, respectively. The parameter $1 - \Psi_f$ is the home bias term in household consumption, Ψ_f is the share of imported core goods in domestic consumption and ϖ represents the intra-temporal elasticity of substitution associated with each group of domestically produced goods and imported foreign goods in the core consumption bundle. The resulting aggregate consumer price index (CPI) is:

$$P_t = \left[(1 - \Psi_{ro})(P_t^c)^{1-\nu} + \Psi_{ro}(P_t^{ro})^{1-\nu} \right]^{\frac{1}{1-\nu}} \quad (3)$$

Where P_t^c and P_t^{ro} are the price of core goods and imported refined oil, respectively. The price index associated with core consumption bundles is as follows:

$$P_t^c = \left[(1 - \Psi_f)(P_t^h)^{1-\vartheta} + \Psi_f(P_t^f)^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}} \quad (4)$$

Where P_t^h and P_t^f represents the price of domestically produced and imported goods, respectively in the core goods consumption basket. The solution of (3) and (4) yields $P_t = (P_t^c)^{1-\Psi_{ro}}(P_t^{ro})^{\Psi_{ro}}$ and $P_t^c = (P_t^h)^{1-\Psi_f}(P_t^f)^{\Psi_f}$, respectively. The total household expenditure [$P_t^c C_t^c + P_t^{ro} C_t^{ro} = P_t C_t$] can be minimized subject to the CES consumption aggregator in (1) to determine household optimal allocation for the aggregate consumption. Thus, the demand for core goods and refined oil in the aggregate consumption are as follows:

$$C_t^c = 1 - \Psi_f \left(\frac{P_t^c}{P_t} \right)^{-\nu} C_t \quad (5)$$

$$C_t^{ro} = \Psi_{ro} \left(\frac{P_t^{ro}}{P_t} \right)^{-\nu} C_t \tag{6}$$

The demand functions for domestically produced and imported goods, resulting from household minimization of core expenditure $P_t^h C_t^h + P_t^f C_t^f = P_t^c C_t^{ci}$ subject to the CES core consumption aggregator in (2) are given as follows:

$$C_t^h = 1 - \Psi_f \left(\frac{P_t^h}{P_t^c} \right)^{-\varpi} C_t^c \tag{7}$$

$$C_t^f = \Psi_f \left(\frac{P_t^f}{P_t^c} \right)^{-\varpi} C_t^c \tag{8}$$

Substituting P_t in 5 and 6; and P_t^c in 7 and 8, respectively; we obtain the following new corresponding sets of demand functions:

$$C_t^c = (1 - \Psi_{ro}) \left(\frac{P_t^{ro}}{P_t} \right)^{\nu \Psi_{ro}} C_t \tag{9}$$

$$C_t^{ro} = \Psi_{ro} \left(\frac{P_t^{ro}}{P_t} \right)^{-\nu(1-\Psi_{ro})} C_t \tag{10}$$

$$C_t^h = 1 - \Psi_f \left(\frac{P_t^f}{P_t^h} \right)^{\varpi \Psi_f} C_t^c \tag{11}$$

$$C_t^f = \Psi_f \left(\frac{P_t^f}{P_t^h} \right)^{-\varpi(1-\Psi_f)} C_t^c \tag{12}$$

Equation 9 is substituted into 11 and 12 to derive the following home and foreign goods consumption equations:

$$C_t^h = (1 - \Psi_f) (\Upsilon_t)^{\varpi \Psi_f} (1 - \Psi_{ro}) \left(\frac{P_t^c}{P_t^{ro}} \right)^{-\nu \Psi_{ro}} C_t \tag{13}$$

$$C_t^f = \Psi_f (\Upsilon_t)^{-\varpi(1-\Psi_f)} (1 - \Psi_{ro}) \left(\frac{P_t^c}{P_t^{ro}} \right)^{-\nu \Psi_{ro}} C_t \tag{14}$$

Where Υ_t is the terms of trade $\left(\frac{P_t^f}{P_t^h} \right)$ as in Gali & Monacelli (2005).

The representative household maximizes an expected inter-temporal utility function of the form:

$$E_0 = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\eta}}{1-\eta} - \frac{L_t^{1+\varrho}}{1+\varrho} \quad (15)$$

Where the expectation operator is E_0 , the discount factor is β^t , a composite consumption goods index is represented as C_t and household hours of work are given as L_t . The relative risk aversion coefficient, a measure of the utility function curvature is represented as η . The elasticity of the marginal dis-utility of labour is ϱ . The household's budget constraint is defined as follows:

$$P_t^{ro} C_t^{ro} + P_t^h C_t^h + P_t^f C_t^f + B_t + \left(F_t \left(\frac{S_t B_t^*}{P_t Y_t}, \epsilon_t^f \right) R_t^* \right)^{-1} S_t B_{t+1}^* + \tau_t \leq S_t B_t^* + (R_t)^{-1} B_{t+1} + W_t L_t + \Pi_t^h \quad (16)$$

Summarizing all the consumption types in the composite consumption bundle of the household, the expression can be re-written as:

$$P_t C_t + B_t + \left(F_t \left(\frac{S_t B_t^*}{P_t Y_t}, \epsilon_t^f \right) R_t^* \right)^{-1} S_t B_{t+1}^* + \tau_t \leq (R_t)^{-1} B_{t+1} + S_t B_t^* + W_t L_t + \Pi_t^h \quad (17)$$

The household has access to one period domestic bond, B_t and one period foreign bond B_t^* at the nominal gross returns⁵ of R_t and R_t^* , respectively. A unit of domestic bond B_t is purchased while a unit of foreign currency denominated $S_t B_t^*$ is issued at time (t); and they attract the gross nominal returns of $(R_t)^{-1} B_t$ and $\left(F_t \left(\frac{S_t B_t^*}{P_t Y_t}, \epsilon_t^f \right) R_t^* \right)^{-1} S_t B_t^*$, respectively at time ($t + 1$). $F_t \left(\frac{S_t B_t^*}{P_t Y_t}, \epsilon_t^f \right)$ is the small open economy's endogenous risk premium associated with private foreign bonds and it is influenced by the ratio of the country's net foreign asset/debt to gross domestic product (GDP) and a stochastic risk premium shock (Garcia & Gonzalez, 2013). A net debtor small open economy issuing foreign debt must pay an extra cost (a risk premium) in addition to a foreign risk-free interest rate. Conversely, if the small open economy is a net creditor, it receives returns less than the foreign risk-free interest rate as foreign bond holders' factor in the country risk premium.

In addition, the inclusion of the risk premium reflects the empirical evidence in sup-

⁵ $R_t = 1 + i$ and $R_t^* = 1 + i^*$ being the gross returns on domestic and foreign bonds, respectively.

port of the existence of an international financial friction or imperfect international capital mobility as in Benigno (2001) in which domestic agents must pay a premium above the foreign risk-free interest rate to access foreign funding. In addition, risk premium on foreign bonds has been shown by Schmitt-Grohe & Uribe (2003) to be an important requirement for inducing stationarity of the small open economy’s total net foreign assets/debts. In a world with perfect international financial market, there will be complete international risk sharing and the risk premium F_t will be equal to unity. S_t is the nominal exchange rate, $W_t L_t$ is wages received from hours of work, Π_t^h is the profits received from ownership of firms producing non-tradable goods and τ_t is the lump-sum tax.

The optimizing household chooses the combination of consumption, C_t , labour supply, L_t , domestic savings, B_t and foreign borrowings, B_t^* , respectively, which maximizes its intertemporal utility in equation (15) subject to the consequent budget constraint in equation (16). The intra-temporal optimality rule for labour supply in the appendix indicates that the optimal labor-leisure decision must be such that the marginal rate of substitution between consumption and hours of work or leisure equates the real wage.

2.3 Domestic Goods Firm

2.3.1 Production

The representative firm produces non-oil final goods for domestic and foreign consumption. By combining labour hours L_t^h and imported refined oil RO_t^h . It seeks to achieve profit maximization by selecting the price of its variety subject to demand functions and available technology. The total output of a given variety, j^h , is $Y_t^h(j^h)$ and the applicable CES production technology can be expressed as:

$$Y_t^h(j^h) = Z_t^h \left[\#_h^{\frac{1}{\omega_h}} (RO_t^h(j^h))^{1-\frac{1}{\omega_h}} + (1 - \#_h)^{\frac{1}{\omega_h}} (L_t^h(j^h))^{1-\frac{1}{\omega_h}} \right]^{\frac{\omega_h}{\omega_h-1}} \tag{18}$$

Where Z_t^h represent productivity variable in the domestically produced goods sector, assumed to evolve as an auto-regressive process, and common to all firms. The imported refined oil RO_t^h and labour employed L_t^h are used to produce the variety j^h .

The weights of refined oil and labour services in production are given by ϑ_h and $1 - \vartheta_h$, respectively, while the degree of factor substitution between refined oil and labour is defined by ω^h . This parameter is crucial in the determination of the significance of the impact of shocks to refined oil on domestic firms' marginal cost, output, and core inflation. The production function for the domestic goods producing firm, in log-linear terms, is given as:

$$\widetilde{y}_t^h = \widetilde{z}_t^h + \#_h \widetilde{ro}_t^h + (1 - \#_h) \widetilde{l}_t^h \quad (19)$$

The optimal inputs mix of firms can be determined by minimizing the firm's total inputs cost subject to technology constraint. This can be expressed as follows:

$$\begin{aligned} \text{Min. : } & W_t L_t^h(j^h) + P_t^{ro} RO_t^h(j^h) \\ \text{s.t : } & Y_t^h(j^h) = Z_t^h \left[\#_h^{\frac{1}{\omega_h}} (RO_t^h(j^h))^{1-\frac{1}{\omega_h}} + (1 - \#_h)^{\frac{1}{\omega_h}} (L_t^h(j^h))^{1-\frac{1}{\omega_h}} \right]^{\frac{\omega_h}{\omega_h-1}} \end{aligned} \quad (20)$$

The FOCs of (20) with respect to labour $L_t^h(j^h)$ and refined oil $RO_t^h(j^h)$ is utilised to derive the nominal marginal cost equation as follows:

$$NMC_t^h = (Z_t^h)^{-\frac{1}{\omega_h}} \left[\#_h (P_t^{ro})^{1-\omega_h} + (1 - \#_h) W_t^{1-\omega_h} \right]^{\frac{1}{1-\omega_h}} \quad (21)$$

where price of imported refined oil with subsidy is expressed as $P_t^{ro} = S_t P_t^{ro*} - \Theta_t$; Θ_t is the subsidy element. The quantity of domestically produced goods sold at home and abroad are represented by $C_t^h(j^h)$ and, $C_t^{h*}(j^h)$ respectively, while the corresponding demand functions for a particular good variety can be expressed as $C_t^h(j^h) = \left(\frac{P_t^h(j^h)}{P_t^h} \right)^{-\epsilon_h} Y_t^h$; $C_t^{h*}(j^h) = \left(\frac{P_t^{h*}(j^h)}{P_t^{h*}} \right)^{-\epsilon_h} Y_t^{h*}$. Where $P_t^h(j^h)$ is the price of domestically produced good variety j^h sold at home and $P_t^{h*}(j^h)$ is the foreign currency price of domestic good variety sold abroad. The parameter ϵ_h represents the

demand elasticity for domestic good variety j^h while P_t^h and P_t^{h*} are the aggregate price indices for the goods variety sold domestically and abroad, respectively.

2.3.2 Domestic Goods Price Setting

Domestic firms are assumed to face monopolistic competition in the home market such that domestic pricing is staggered a la Calvo (1983); whereas pricing of the export component of the domestically produced variety is perfectly competitive⁶. Domestically, a fraction of the firms with the probability of $1 - \theta^h$ receives a price adjustment signal at time t and thus re-optimizes at $t + 1$ while another fraction with the probability θ^h are stuck with the previous period's price because they do not receive the signal for price reset⁷. Thus $\theta^h \in (0, 1)$ represents a measure of price stickiness or nominal rigidity associated with the pricing for home produced goods. The fraction $\frac{1}{1-\theta^h}$ represents the period domestic goods prices are expected to remain inflexible. In addition, we assume that firms that can re-optimize⁸ every period are in two categories, namely: “forward-looking and backward-looking firms”. The forward-looking firms adjust prices optimally using all the information at their disposal at the time of decision making. Backward-looking firms on the other hand, depend on a rule of thumb for setting prices. They assume information available to them is sticky, consequently they extract and process such information with delay and utilize their knowledge about the historical evolution of price levels to set prices. To reset their prices $P_t^h(j^h)$, backward-looking firms index current prices to previous period inflation. The index of domestic prices is given as:

$$P_t^h(j^h) = P_{t-1}^h(j^h) \left(\frac{P_{t-1}^h}{P_{t-2}^h} \right)^{\theta^h} \tag{22}$$

⁶All goods for exports are competitively priced.

⁷Price stickiness exists in the short-run due to staggered prices, menu cost, coordination failure, etc (Snowdon & Vane, 2005 and Junior, 2016)

⁸Reset their prices.

The aggregate domestic price index can be expressed as:

$$P_t^h = (1 - \theta^h) \left(\overset{reset}{P_t^h} \right)^{1-\epsilon_h} + \theta^h \left[P_{t-1}^h \left(\frac{P_{t-1}^h}{P_{t-2}^h} \right)^{\theta^h} \right]^{1-\epsilon_h \frac{1}{1-\epsilon_h}} \quad (23)$$

Where $\frac{1}{1-\epsilon_h}$ is the mark-up and $\overset{reset}{P_t^h}(j^h)$ is the new price set by an optimizing domestic firm producing good j^h variety to maximize the present discounted value of expected future profits stream. After a series of profit optimisation procedures as in Gali (2008), the log-linearized New Keynesian hybrid Philips Curve is obtained as:

$$\widetilde{\pi}_t^h = (1 - \beta\theta^h) E_t(\widetilde{\pi}_{t+1}^h) + \theta^h \widetilde{\pi}_{t-1}^h + \kappa_t^h \widetilde{rmc}_t^h, \quad (24)$$

where $\kappa_t^h = \frac{(1-\beta\theta^h)(1-\theta^h)}{\theta^h}$, is the coefficient of marginal cost. The hybrid New Keynesian Philips Curve in (24) indicates that inflation is determined by the forward-looking $E_t(\widetilde{\pi}_{t+1}^h)$, and the backward-looking $\widetilde{\pi}_{t-1}^h$ inflation elements, as well as the firm's real marginal cost.

2.4 Imports Price Setting and Incomplete pass-through

A representative retailer imports final core good from the rest of the world and applies a mark-up on the foreign price. This gives rise to a wedge between the foreign (P_t^{f*}) and domestic (P_t^f) import prices. This leads to law of one price gap ψ_t (Monacelli, 2005). In addition, following empirical evidence, Burstein & Gopinath (2014) observed that the pass-through from import prices to domestic prices is somewhat subdued; thus, generating a deviation from the law of one price. The law of one price gap has been defined as follows:

$$\psi_t = \frac{S_t P_t^{f*}}{P_t^f} \quad (25)$$

Analogous to the pricing for domestic goods, the import retailer faces a downward

sloping demand curve for imported goods of the form:

$$C_{j,t}^f = \Psi_f \left(\frac{P_{j,t}^f}{P_t^c} \right)^{-\epsilon_f} C_t^c \tag{26}$$

Import good pricing is subject to Calvo (1983)’s staggered pricing mechanism. A ratio $1 - \theta^f$ of importing firms can re-optimize while the ratio θ^f cannot change their price. Among importers that can re-optimize, a group is “backward-looking” and the other “forward-looking”. The imports hybrid New Keynesian Phillip’s curve which expresses the average imported inflation as a function of the expected import inflation, lagged import inflation and the law of one price gap is derived as follows:

$$\widetilde{\pi}_t^f = (1 - \beta\theta^f) E_t(\widetilde{\pi}_{t+1}^f) + \theta^f \widetilde{\pi}_{t-1}^f + \kappa_t^f \widetilde{\psi}_t \tag{27}$$

Where $\widetilde{\psi}_t$ is the law of one price gap and is equivalent to the real marginal cost RMC_t of imports. If $\widetilde{\psi}_t \equiv 1$, then law of one price holds, implying that the foreign price of imports p_t^{f*} is equal to domestic currency price of imports p_t^f . θ^f is the import price stickiness parameter and $\kappa_t^f = \frac{(1-\beta\theta^f)(1-\theta^f)}{\theta^f}$ is the coefficient of the law of one price gap.

2.5 Real Exchange Rate, Terms of Trade, and Foreign Demand

The real exchange rate Q_t is defined as follows:

$$Q_t = \frac{S_t P_t^{f*}}{P_t} \tag{28}$$

It is the ratio of the aggregate foreign price index to the aggregate domestic price index. An increase in the ratio implies depreciation and appreciation, otherwise. The terms of trade, Υ_t , is expressed as follows:

$$\Upsilon_t = \frac{P_t^f}{P_t^h} = \frac{S_t P_t^{f*}}{P_t^h} \tag{29}$$

Υ_t is the price of imports relative to price of domestically produced goods. The

domestic real price of imported refined oil can be expressed as:

$$\frac{P_t^{ro}}{P_t} = Q_t \frac{P_t^{ro*}}{P_t^{f*}} \quad (30)$$

Where $P_t^{ro} = S_t P_t^{ro*}$ is the domestic nominal price of refined oil, $P_t^{f*} \equiv P_t^*$ is the foreign price index and P_t^{ro*} is the foreign price of refined oil. The small open economy exports domestically produced goods and crude oil to the rest of the world. The foreign demand for domestically produced goods by the rest of the world is:

$$C_t^{h*} = \Psi_h^* \left(\frac{P_t^{h*}}{P_t^{f*}} \right)^{-\epsilon_h^*} C_t^*, \quad (31)$$

where ϵ_h^* is the foreign price elasticity of demand for domestically produced goods, Ψ_h^* is the share of domestically produced core goods in foreign households' consumption basket. The export sector is modeled based on the assumption of the law of one price, implying a complete pass-through from domestic to foreign prices for exports. Consequently, C_t^{h*} is priced in foreign market as $P_t^{h*} = \frac{P_t^h}{S_t}$, and when P_t^{h*} is substituted in (31) and re-arranged such that foreign demand for the SOE's goods is expressed as a function of the real exchange, Q_t equation (32) results:

$$\begin{aligned} C_t^{h*} &= \Psi_h^* \left(\frac{P_t^h}{S_t P_t^{f*}} \right)^{-\epsilon_h^*} C_t^*, C_t^{h*} = \Psi_h^* \left(\frac{P_t^h}{P_t} \right)^{-\epsilon_h^*} \left(\frac{P_t}{S_t P_t^{f*}} \right)^{-\epsilon_h^*} C_t^*, \\ C_t^{h*} &= \Psi_h^* \left(\frac{P_t^h}{P_t} \right)^{-\epsilon_h^*} \left(\frac{1}{Q_t} \right)^{-\epsilon_h^*} C_t^* \end{aligned} \quad (32)$$

The small open economy assumption underpinning the model implies that the share of domestically produced goods Ψ_h^* in foreign consumption basket C_t^* is negligible and preferences between domestic and foreign consumers are symmetric.

2.6 Imperfect International Risk Sharing and the Uncovered Interest Rate Parity

Foreign households are required to solve inter-temporal consumption, savings, and labour supply problems analogous to that of domestic households, except that domes-

tic households face an endogenous risk premium, $F_t \left(\frac{S_t B_t^*}{P_t Y_t}, \epsilon_t^F \right)$, when they participate in the international financial market to smooth consumption. Under this condition, consumption risk is not perfectly shared between domestic and foreign households. Hence, the need to augment the equality between domestic and foreign consumption Euler equations with the risk premium, $F_t \left(\frac{S_t B_t^*}{P_t Y_t}, \epsilon_t^F \right)$, as follows:

$$\frac{1 + i_t}{(1 + i_t^*) F_t \left(\frac{S_t B_t^*}{P_t Y_t}, \epsilon_t^F \right)} = \frac{E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\eta} \left(\frac{P_t}{P_{t+1}} \right) \right]}{E_t \left[\left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\eta} \left(\frac{S_t P_t^*}{S_{t+1} P_{t+1}^*} \right) \right]} \tag{33}$$

Equation (33) can be solved iteratively in line with Gali & Monacelli (2005), to derive the following expression:

$$C_t = \Omega (Q_t F_t)^{\frac{1}{\eta}} C_t^* \tag{34}$$

where Ω is a constant, representing the initial assets position, C_t is domestic consumption, and C_t^* is foreign consumption. From (34), the ratio of marginal utility of consumption to price between domestic and foreign consumers is not equal, resulting to a relative demand gap⁹ shown by $F_t \neq 1$. This underlies the risk sharing incompleteness between domestic and foreign households in the model. The uncovered interest rate parity (UIP) condition which shows the no-arbitrage condition in an incomplete international financial market situation can be derived as follows:

$$\frac{1}{R_t^* F_t} = \left(\frac{1}{R_t} \right) \left(\frac{S_t}{E_t S_{t+1}} \right) \tag{35}$$

In equation (35), the presence of F_t alters the conventional interest rate parity condition into a modified version which suggests that the real interest rate differential between the SOE and the foreign economy is accounted for by the expected exchange rate depreciation and the risk premium.

2.7 External Debt and the Risk Premium

We assume both households and the government participate in the domestic bonds

⁹ As discussed in Motyovszki (2016) and Jalali Naini & Naderian (2017)

market, while only households can access the foreign bonds market. Domestic bonds holding dynamics results to a zero net supply¹⁰, such that $B_t = 0$. For simplicity, we assume that households do not hold foreign bonds, but issue bonds to foreigners. Consequently, the small open economy is a net borrower, such that foreign bonds holding B_t^* is positive and the household pays a premium on top of a foreign risk-free interest rate. In the circumstance, if the SOE were to be a net lender, it would earn less returns on bonds purchased from foreign issuers.

Following Schmitt-Grohe & Uribe (2003), Cavoli (2009), Garcia & Gonzalez (2013), Motyovszki (2016) and Kreptsev & Seleznev (2018), the risk premium, F_t , is influenced by the deviation of foreign debt to GDP ratio d_t (i.e. $\frac{S_t B_t^*}{P_t Y_t}$) from its steady value d in log-linear form and a stochastic risk premium shock ϵ_t^F . The risk premium shock is modelled as a second source of external vulnerability in addition to the negative oil price shock, for the small open economy. This follows Smets & Wouters (2007) and Smets *et al.* (2010) who reports that the domestic risk premium shock act like a negative demand shock, leading to extended decline in domestic spending. In addition, a positive risk premium shock increases the cost of foreign bonds issuance; thereby constraining domestic households' ability to acquire foreign debt to smooth consumption in the event of an adverse shock. The risk premium equation can be formalized in log-linear form as follows:

$$\tilde{F}_t = \Phi_d \tilde{d}_t + \epsilon_t^F \quad (36)$$

Where Φ_d is the elasticity parameter with respect to foreign debt to GDP ratio while ϵ_t^F is the stochastic risk premium shock, assumed to approximate other influences on the risk premium. The risk premium is an increasing function of the economy's external debt to GDP ratio.

2.8 The Domestic Oil Sector

The small open economy is endowed with oil reserves over which the government exercises control. The representative oil firm obtains a government license and pays taxes to government. Crude oil output of the representative oil firm is exported

¹⁰ The domestic bonds market is always in equilibrium. Every amount borrowed by one agent (say households) is saved by the other (say the government) and vice versa.

wholly to the rest of the world. The world oil market is characterized by perfect competition; with the price of crude oil in domestic currency given as $P_t^{co} = S_t P_t^{co*}$, and it evolves exogenously as follows:

$$\widetilde{P}_t^{co*} = \rho_{P^{co*}} \widetilde{P}_{t-1}^{co*} + \varepsilon_t^{P^{co*}} \quad \varepsilon_t^{P^{co*}} \sim N(0, \sigma_{P^{co*}}^2) \quad (37)$$

Crude oil production Y_t^{co} requires labour and oil capital mix. This is expressed using a Cobb-Douglas type production function in (38), where Z_t^{co} is oil sector productivity, K_t^{co} is the previous period oil capital, L_t^{co} is labour supply in the oil sector and γ_{co} represents the share of oil capital in oil production. The representative oil firm maximizes net oil revenue¹¹ and solves the following maximization problem:

$$Max. : NR_t^{co} = [GR_t^{co} - W_t L_t^{co} - R_t^{co} K_t^{co}]; \text{ s.t.: } Y_t^{co} = Z_t^{co} (K_{t-1}^{co})^{\gamma_{co}} (L_t^{co})^{1-\gamma_{co}} \quad (38)$$

where NR_t^{co} is net crude oil revenue, the revenue accruing to the representative oil firm after capital and labour costs have been deducted from gross crude oil revenue, GR_t^{co} (which is $P_t^{co} Y_t^{co}$). Net crude oil revenue, NR_t^{co} , is subject to tax¹² τ^{co} from the government and the balance $1 - \tau^{co}$ is paid as returns to foreign investors who provide oil sector capital in form of foreign direct investment (FDI_t). The law of motion for oil sector capital is as follows:

$$K_{t+1}^{co} = (1 - \delta_{co}) K_t^{co} + FDI_t \quad (39)$$

Given that oil output Y_t^{co} is exported wholly to the rest of the world, such that $Y_t^{co} = C_t^{hco*}$, the foreign demand for the SOE's crude oil is as follows:

$$C_t^{hco*} = \Psi_{co}^* \left(\frac{P_t^{co*}}{P_t^{f*}} \right)^{-\epsilon_{co}^*} C_t^{co*} \equiv C_t^{hco*} = \Psi_{co}^* \left(\frac{P_t^{co}}{P_t} \right)^{-\epsilon_{co}^*} \left(\frac{1}{Q_t} \right)^{-\epsilon_{co}^*} C_t^{co*}. \quad (40)$$

The foreign price elasticity of demand for domestically produced crude oil is given

¹¹The representative oil producing firm is, in effect, trying to maximize government revenues since it does not make profit.

¹²This captures royalties and petroleum profit tax.

by ϵ_{co}^* and the share of domestically produced crude oil in total foreign crude oil consumption is given by Ψ_{co}^* . Given our small open economy assumption, the share of domestically produced crude oil in total foreign oil consumption basket C_t^{co*} is negligible. Both foreign crude oil price P_t^{co*} and foreign oil consumption C_t^{co*} are assumed to evolve exogenously.

2.9 The Labour Market

A competitive labour market is assumed, ensuring wage equality across the two production sectors. The two wage equations were derived from first order conditions of the two production functions with respect to labour as: $W_t^h = NMC_t^h \left(\frac{(1-\#_h) Y_t^h}{L_t^h} \right)^{\frac{1}{\omega_h}}$ and $W_t^{co} = (1 - \gamma_{co}) \left(\frac{P_t^{co} Y_t^{co}}{L_t^{co}} \right)$. Equalizing the two wage expressions, yields:

$$W_t = \left(\frac{(NMC_t^h) L_t^{co}}{(1 - \gamma_{co}) (P_t^{co} Y_t^{co})} \right) \left(\frac{(1 - \#_h) Y_t^h}{L_t^h} \right)^{\frac{1}{\omega_h}} \quad (41)$$

2.10 Government

Government consumes both domestically produced and imported goods. It undertakes the importation of refined oil and sells it to households at a subsidized price. Thus, refined oil subsidy constitutes parts of government expenditure. The quantum of subsidy depends on foreign oil market dynamics and the degree of its intervention in the sector. The government consumption basket includes domestically produced goods and imports as follows:

$$G_t = \left[(1 - \Psi_g)^{\frac{1}{v_g}} (G_t^h)^{\frac{v_g-1}{v_g}} + (\Psi_g)^{\frac{1}{v_g}} (G_t^f)^{\frac{v_g-1}{v_g}} \right]^{\frac{v_g}{v_g-1}} \quad (42)$$

Where Ψ_g and $1 - \Psi_g$ are weights of imported and domestic goods, respectively, in government consumption basket. v_g is the elasticity of substitution between the two goods variety in government consumption bundle. Applying the CES consumption aggregator and minimizing expenditure, government demand functions for the two goods categories becomes:

$$G_t^h = 1 - \Psi_g \left(\frac{P_t^h}{P_t^c} \right)^{-v_g} G_t; G_t^f = \Psi_g \left(\frac{P_t^f}{P_t^c} \right)^{-v_g} G_t \quad (43)$$

The price index for government consumption expenditure is:

$$P_t^c = \left[(1 - \Psi_g)(P_t^h)^{1-v_g} + \Psi_g(P_t^f)^{1-v_g} \right]^{\frac{1}{1-v_g}} \tag{44}$$

Consequently, the small open economy’s government minimum total consumption expenditure is $P_t^h G_t^h + P_t^f G_t^f = P_t^c G_t$ and fiscal behaviour is assumed to follow an exogenous process: $\tilde{g}_t = \rho_g \tilde{g}_{t-1} + \epsilon_t^g$ $\epsilon_t^g \sim N(0, \sigma_g^2)$. Government revenue is sourced through receipts from lump-sum tax τ_t levied on households and tax on oil firm’s net crude oil proceeds $\tau^{co} NR_t^{co}$ and expenditure includes government spending on domestic and foreign goods $P_t^h G_t^h$ and $P_t^f G_t^f$, respectively; and refined oil subsidy¹³ payments. Subsidy payments ($\Theta_t M_t^{ro}$) is the product of the differential between foreign and domestic pump price of imported fuel, Θ_t and the total quantity of imported refined oil, M_t^{ro} . Government budget constraint is expressed as follows:

$$\tau^{co} \left(\frac{NR_t^{co}}{P_t} \right) + \tau_t = G_t^h + G_t^f + \Theta_t M_t^{ro} \tag{45}$$

Where $M_t^{ro} = C_t^{ro} + RO_t^h$ with C_t^{ro} and RO_t^h being refined oil consumed by households and domestic firms, respectively. Where τ_t evolves log-linearly as follows:

$$\tilde{\tau}_t = \rho_\tau \tilde{\tau}_{t-1} + (1 - \rho_\tau) \left(\chi_1 \tilde{d} - \chi_2 (\tilde{p}_t^{co} + \tilde{y}_t^{co} - \tilde{p}_t) \right) + \epsilon_t^\tau \tag{46}$$

The expression for lump-sum tax includes a smoothing component (ρ_τ) and its sensitivity to the economy’s debt to GDP ratio (χ_1) and crude oil revenues (χ_2). Many emerging and developing oil producing economies tend to intensify tax efforts whenever the threat of a debt overhang looms, with external borrowing conditions become tighter; while they tend to relax their non-oil tax revenue efforts whenever the oil sector is booming (Tijerina-Guajardo & Pagán, 2003).

2.11 Refined Oil Price

We model imported refined oil price in the spirit of Bouakez *et al.* (2008) and Allegret & Benkhodja (2015), as a convex combination of the immediate past period’s

¹³Government pays the difference between the foreign price of refined oil and the domestic price.

domestic price and the prevailing foreign price of refined oil expressed in domestic currency as follows:

$$P_t^{ro} = \varsigma P_{t-1}^{ro} + (1 - \varsigma) S_t P_t^{ro*} \quad (47)$$

Where $\varsigma \in (0,1)$ is the oil subsidy indicator and can vary based on the extent to which government is intervening in the downstream sector of the oil industry. The foreign price of refined oil is $S_t P_t^{ro*}$ while the domestic pump price of refined oil is P_t^{ro} . If the subsidy indicator parameter $\varsigma = 1$, the degree of intervention is total and there is no pass-through from foreign price of refined oil to the domestic price; ensuring that the domestic price is fixed at the old level and the differential is completely picked up by government subsidy payment, $\Theta_t M_t^{ro}$. If, however, $\varsigma = 0$, then there is no subsidy, domestic refined oil price will reflect fully foreign dynamics of refined oil price and as such there will be a complete pass-through. We will consider three (3) subsidy experiments, namely: (a) full subsidy, (b) zero subsidy and (c) partial subsidy at a fraction of 0.5.

The refined oil pricing rule, though arbitrary since it is not derived from an explicit optimization of government behaviour; it is, however, used to reflect the practice in many emerging small open economies, where domestic price of refined oil prices are based on ad hoc pricing templates intended to smooth oil price volatility. The pricing rule is also useful for showing that fiscal intervention through oil subsidy may not permit complete oil price pass-through. Also, the fiscal commitment to an oil pricing rule may be a good anchor for agents' expectations about refined oil price (Bouakez *et al.*, 2008). The equation for the differential between foreign and pump price of imported fuel is as follows:

$$\Theta_t = S_t P_t^{ro*} - P_t^{ro} \quad (48)$$

The oil pricing rule can potentially generate an increase or a decrease in oil subsidy depending on the nature of the oil price shock and the size of the subsidy parameter. We propose a relationship to account for the transmission dynamics between foreign crude oil price and foreign refined oil price based on empirical evidence¹⁴. This

¹⁴Data obtained from the US Energy Information Administration

relationship evolves log-linearly as follows:

$$\widetilde{p}_t^{ro*} = \zeta_{p^{co*}} \widetilde{p}_{t-1}^{co*} + (1 - \zeta_{p^{ro*}}) \widetilde{va}_t^{ro*} + \epsilon_t^{ro*} \tag{49}$$

Equation 50 show that foreign price of refined oil depends on last period’s foreign price of crude oil p_{t-1}^{co*} and the current period’s value-added costs va_t^{ro*} , which may include refining cost, foreign tax on refined oil and distribution or marketing costs. The relationship between previous period foreign price of crude oil and the current period foreign price of refined oil is governed by $\zeta_{p^{co*}}$ while ϵ_t^{ro*} is an exogenous shock. This pricing rule establishes a nexus between foreign prices of crude oil and refined oil. The characterization can assist a net oil exporting developing economy to gauge the net real value derivable from its oil endowments and underscore the need to maximize the benefits oil domestically. Such a country too, can gain insight on the extent to which it is vulnerable to oil-related adverse shocks. Oil refining value added cost, va_t^{ro*} , evolve exogenously; $\widetilde{va}_t^{ro*} = \rho_{va^{ro*}} \widetilde{va}_{t-1}^{ro*} + \epsilon_t^{va^{ro*}}$.

2.12 Aggregation and Market Clearing

To satisfy the aggregate market clearing condition, the country’s gross domestic product (Y_t) equals domestic output plus exports less imports ($Y_t = Y_t^h + X_t - M_t$). The summary of key aggregation equations is as follows:

Aggregate output: $Y_t = Y_t^h + X_t - M_t$

Output of domestically produced core goods: $Y_t^h = C_t^h + C_t^{h*} + G_t^h$

Output of crude oil: $Y_t^{hco} = C_t^{hco*}$

Aggregate import: $M_t = M_t^f + M_t^{ro}$

Core import: $M_t^f = C_t^f + G_t^f$

Refined oil import: $M_t^{ro} = C_t^{ro} + RO_t^h$

Export: $X_t = C_t^{h*} + Y_t^{hco}$

Aggregate bonds sum up domestic and foreign bonds ($B_t^{agg} = B_t + B_t^*$). Domestic bonds are assumed to be in net zero supply such that, $B_t = 0$ and foreign bonds B_t^* are issued by domestic households while domestic households do not subscribe to foreign issued bonds. Our bonds clearing condition, thus, equate aggregate bonds holding to total foreign bonds holding ($B_t^{agg} = B_t^*$).

The labour market aggregation results from the sum of employment in the non-oil

and oil sectors ($L_t = L_t^h + L_t^{co}$).

The current account equates foreign debt service adjusted by the country's risk premium¹⁵ to the sum of total trade balance and foreign debt holdings. This can be presented as follows:

$$\left(F_t \left(\frac{S_t B_t^*}{P_t Y_t}, \epsilon_t^F \right) R_t^* \right) S_t B_{t+1}^* = S_t B_t^* + P_t^x X_t - P_t^m M_t \quad (50)$$

Where $P_t^x X_t$ and $P_t^m M_t$ are the nominal exports and imports, respectively. These identities are further defined as follows:

$$P_t^x X_t = S_t P_t^{h*} C_t^{h*} + S_t P_t^{co*} Y_t^{hco*} \quad (51)$$

$$P_t^m M_t = P_t^f C_t^f + P_t^g G_t^f + P_t^{ro} C_t^{ro} + P_t^{ro} R O_t^h \quad (52)$$

Equation (50) expressed in terms of B_t^* in log-linear form is:

$$\widetilde{b}_t^* = \frac{1}{\beta} \widetilde{b}_{t-1}^* + \widetilde{n x}_t - f \widetilde{dir}_t + \frac{\overline{NX/PY}}{\beta - 1} \left(\widetilde{r}_{t-1}^* + \widetilde{F}_{t-1} + \Delta \widetilde{q}_t - \widetilde{\pi}_t \right) \quad (53)$$

Where $f \widetilde{dir}_t$ is the returns received by foreign direct investors in the oil sector and it evolves according to the rule, $f \widetilde{dir}_t = 1 - \tau^{co} (nr_t^{co})$; and the log-linear net export is $\widetilde{n x}_t = \widetilde{x}_t - \widetilde{m}_t$.

2.13 Monetary Policy

The model is closed with a Taylor (1993)-type interest rate rule augmented with the exchange rate. The exchange rate is under some degree of monetary policy watch in many emerging SOEs. The central bank is assumed to react to deviation of output, inflation, and the exchange rate from their steady state values. It is common in the literature to include the exchange rate in the monetary policy rules for small open economy models¹⁶. The log-linear expression for the generalized Taylor rule in (54), feature three important inflation variants, namely: aggregate inflation π_t , domestic

¹⁵Although we assume foreign bonds issuance is limited to domestic households, government liability is implied as these bonds are often backed by the full faith of government.

¹⁶See Gali & Monacelli (2005), De Paoli (2009), Senbeta (2013), Hove et al. (2015), Allegret & Benkhodja (2015).

inflation π_t^h and core inflation π_t^c in addition to output and the exchange rate.

$$\widetilde{i}_t = \rho_i \widetilde{i}_{t-1} + (1 - \rho_i) (\$1 \widetilde{y}_t + \$2 \widetilde{\pi}_t + \$3 \widetilde{\pi}_t^c + \$4 \widetilde{\pi}_t^o + \$5 \widetilde{s}_t) + \epsilon_t^i \tag{54}$$

Where $\varpi_1, \varpi_2, \varpi_3, \varpi_4$ and ϖ_5 are weights associated with output, aggregate inflation, core inflation, oil inflation and the exchange rate, respectively in the generalized Taylor rule. The size of each weight reflects the relative importance of the corresponding variables in the monetary policy reaction function. ρ_i is the smoothing parameter and it captures policy inertia or policy history dependence (Woodford, 2003) while ϵ_t^i is the monetary policy shock. From the generalized Taylor rule in (54), four alternative policy rules are formulated, including aggregate inflation targeting rule, core inflation targeting rule, oil inflation targeting rule and real exchange rate targeting rule, respectively. Under all the four policy regimes, the policy maker exhibit appetite for interest rate smoothing while monitoring aggregate output developments; and combines these with one of aggregate inflation (headline), domestic inflation or core inflation.

2.14 The Foreign Economy

Foreign variables evolve exogenously. $\widetilde{i}_t^* = \rho_{i^*} \widetilde{i}_{t-1}^* + \epsilon_t^{i^*}$ is for foreign interest rate, $\widetilde{c}_t^* = \rho_{c^*} \widetilde{c}_{t-1}^* + \epsilon_t^{c^*}$ for foreign consumption (demand), $\widetilde{c}_{t-1}^{co*} = \rho_{c^{co*}} \widetilde{c}_{t-1}^{co*} + \epsilon_t^{c^{co*}}$ for foreign crude oil demand, $\widetilde{\pi}_t^* = \rho_{\pi^*} \widetilde{\pi}_{t-1}^* + \epsilon_t^{\pi^*}$ for foreign inflation, $\widetilde{p}_t^{f*} = \rho_{p^{f*}} \widetilde{p}_{t-1}^{f*} + \epsilon_t^{p^{f*}}$ for foreign imports price, and $\widetilde{p}_t^{co*} = \rho_{p^{co*}} \widetilde{p}_{t-1}^{co*} + \epsilon_t^{p^{co*}}$ for foreign crude oil price.

3. Parameters, Solution and Simulation

The general equilibrium solution of the model is characterized by the sequence of equilibrium conditions satisfying economic agents’ first order conditions, market clearing conditions, the monetary policy rule, the refined oil pricing rule, the lump-sum tax rule, the government budget constraint, the external debt equation, and the risk premium equation. The model solution is up to a second-order approximation around a deterministic steady state level where all variables are constant. Parameter values are calibrated in line with standard small open economy DSGE literature while steady state ratios match stylized features of a net oil exporting emerging economy. Table 1 below contain parameters characterizing the model. Parameter values are in

line with standard literature on small open economies(Taylor, 1993; Schmitt-Grohe & Uribe, 2003; Romero, 2008; Mishkin, 2007; Taylor, 2001; Ortiz & Sturzenegger, 2007; Steinbach *et al.*, 2009; Alpanda *et al.*, 2010; Santacreu, 2005; Hove *et al.*, 2015; Christiano *et al.*, 2005; Gali & Monacelli, 2005; Hove *et al.*, 2015; Zeufack *et al.*, 2016; and Omotosho, 2019).

Table 1: Model parameters and values

Parameter	Description	Value
Ψ_{ro}	Share of refined oil in HHs consumption	0.25
ν	Elasticity of subst. btw. core and oil consumption	0.2
β	Discount Factor	0.96
ϖ	Elasticity of subst. within group in core consumption	8
Ψ_f	Share of imported core goods in HHs consumption	0.35
η	Relative risk aversion coefficient	1
ϱ	Elasticity of the marginal dis-utility of labour	6
ω^h	Domestic firm's factor subst. btw. refined oil and labour	0.5
ϑ_h	Refined oil weight in domestic firm's production function	0.3
ϵ_h	Elasticity of demand for domestic good variety	1
θ^h	Stickiness parameter for domestic good	0.75
θ^f	Stickiness parameter for imports	0.6
ϵ_h^*	Foreign elasticity of demand for domestic goods	1
$\rho_{p^{cos}}$	Foreign crude oil price persistence	0.88
γ_{co}	Capital share in crude oil production	0.65
δ_{co}	Oil capital depreciation rate	0.025
ϵ_{co}^*	Foreign elasticity of demand for SOE's crude oil	1
ν_g	Elasticity of subst. for govt. consumption variety	10
Ψ_g	Core imports weight in government consumption	0.35
$\rho_{z^{co}}$	Crude oil productivity shock persistence	0.85
ρ_{z^h}	Domestic good productivity shock persistence	0.8
	Full subsidy indicator	1
ζ	Partial subsidy indicator	0.5
	Zero subsidy indicator	0
ρ_τ	Lump sum tax shock persistence	0.85

Table 1: Cont'd

Parameter	Description	Value
χ_1	Tax sensitivity to foreign debt GDP ratio	0.6
χ_2	Tax sensitivity to crude oil revenue	0.95
$\rho_{\nu a^{ro*}}$	Foreign refined oil value added cost shock persist.	0.75
Φ_d	Risk premium elasticity wrt. debt/GDP ratio	0.0001
ρ_{p^f*}	Foreign core goods price shock persistence	0.85
ρ_i	Taylor Rule smoothing parameter	0.75
ϖ_1	Output weight in the Taylor rule	0.5
ϖ_2	CPI Inflation weight in Taylor rule	1.5
ϖ_3	Core Inflation weight in the Taylor rule	1.5
ϖ_4	Oil Inflation weight in the Taylor rule	1.5
ϖ_5	Exchange rate weight in the Taylor rule	0.25
ρ_{i^*}	Foreign Interest rate shock persistence	0.85
ρ_{c^*}	Foreign core consumption shock persistence	0.9
$\rho_{c^{co*}}$	Foreign oil consumption shock persistence	0.85
ρ_{π^*}	Foreign inflation shock persistence	0.85
$\zeta_{p^{co*}}$	Refined oil price dependence on past crude oil price	0.9
τ^{co}	Tax on net crude oil revenue	0.75
ρ_{fdi}	FDI persistence parameter	0.55

The steady state ratios in Table 2 are calibrated based on Nigerian data¹⁷ sourced from the International Financial Statistics (IFS), the Nigerian Bureau of Statistics (NBS), and the Central Bank of Nigeria (CBN).

The model is solved in Dynare, utilizing the Blanchard & Khan (1980) procedure and we simulate a 10 percent negative oil price shock before performing an optimization exercise on the coefficients of the alternative Taylor rule specifications under three subsidy or pass-through assumptions subject to the model’s equilibrium conditions.

¹⁷The country is a typical example of a net oil exporting country described in our model setup.

Table 2: Model's Steady States Ratios and Values

Ratio	Description	Value
$\frac{C}{Y}$	Ratio of domestic consumption to total output	0.85
$\frac{G^h}{Y^h}$	Ratio of government consumption of domestic goods to total	0.05
$\frac{X}{Y}$	Ratio of total exports to total output	0.15
$\frac{M}{Y}$	Ratio of total imports to total output	0.1
$\frac{C^{hs}}{X}$	Ratio of core export to total exports	0.1
$\frac{C^{hcos}}{X}$	Ratio of crude oil exports to total exports	0.9
$\frac{M^f}{M}$	Ratio of core imports to total imports	0.85
$\frac{M^{ro}}{M}$	Ratio of refined oil imports to total imports	0.15
$\frac{C^f}{M^f}$	Household share of core imports consumption	0.85
$\frac{G^f}{M^f}$	Government share of core imports consumption	0.15
$\frac{M^f}{C^{ro}}$	Household share of imported refined oil to total	0.75
$\frac{M^{ro}}{M^f}$	Firms share of imported refined oil to total	0.25
$\frac{\Theta M^{ro}}{G}$	SS ratio of oil subsidy to total government expenditure	0.45
$\frac{\tau^{co} NR^{co}}{G}$	SS ratio of govt. net crude oil revenue to govt. expenditure	0.9
$\frac{\bar{\tau}}{G}$	SS ratio of lump-sum tax to government expenditure	0.1
$\frac{G^h}{G}$	SS ratio of gov. spending on home goods to total govt. exp.	0.3
$\frac{G^f}{G}$	SS ratio of gov. spending on imports to total govt. exp.	0.25
$\frac{L^h}{L}$	SS ratio of non-oil sector employment to total employment	0.85
$\frac{L^{co}}{L}$	SS ratio of oil sector employment to total employment	0.15
$\frac{NX}{Y}$	SS ratio of net exports to total output	0.05
$\frac{NR^{co}}{GR^{co}}$	SS ratio of net crude oil revenue to gross crude oil revenue	0.8
$\frac{WL^{co}}{GR^{co}}$	SS ratio of oil sector labour cost to gross crude oil revenue	0.05
$\frac{R^{co} K^{co}}{GR^{co}}$	SS ratio of oil sector capital cost to gross crude oil revenue	0.15
$\frac{FDI}{K^{co}}$	SS ratio of FDI to oil capital	0.45
$\bar{d} = \frac{SB^*}{PY}$	SS ratio of external debt to GDP	0
$\frac{\Theta M^{ro}}{G}$	SS ratio of refined oil subsidy to total gov. expenditure	0.45
$\frac{QP^{ro*}}{\Theta}$	SS ratio of unregulated real refined oil price to subsidy	3.33
$\frac{RP^{ro}}{\Theta}$	SS ratio of subsidized refined oil price to subsidy	2.33

4. Results

A negative crude oil price shock is simulated under three alternative fiscal regimes and four possible monetary policy rules popular in policy circles. The first fiscal regime implements full oil subsidy, which does not allow any pass-through from for-

eign oil price to the domestic price. The second fiscal regime implements partial oil subsidy, allowing for incomplete pass-through to domestic price of refined oil; while the third regime does not allow for any oil subsidy, thus permitting a complete pass-through from foreign price of refined oil. The response of the economy under alternative fiscal arrangement regarding the subsidy is tested under the four alternative monetary policy settings, namely: (i) headline inflation targeting, (ii) core inflation targeting, (iii) oil inflation targeting; and (iv) exchange rate targeting. The shock is expected to set off responses from twenty macroeconomic variables, including: headline inflation, core inflation, oil inflation, risk premium, aggregate consumption, aggregate output, crude oil output, non-oil output, interest rate, real exchange rate, government consumption, refined oil consumption, refined oil in domestic production, external debt-to-GDP ratio, non-oil employment, oil employment, oil capital, foreign direct investment, real wage and fuel subsidy payments over a 40 period horizon. The impulse responses of the variables are presented and analyzed following the negative oil price shock. The macroeconomic responses depicted by the various impulse-response functions will reflect the dynamic sensitivity of the economy to the shock, under different monetary and fiscal policy considerations.

4.1 Oil Price shock under full subsidy regime

Impulse responses generated following the impact of a 10 standard deviation negative oil price shock on selected macroeconomic variables under four alternative monetary policy rules, assuming full subsidy regime are presented in Figures 2a and 2b.

A full subsidy regime assumes a fiscal arrangement where government insulates the domestic economy from fluctuations in the international price of oil. Under this arrangement, households and firms will not be impacted by the shock. As a result, agents will be indifferent to both positive and negative movements in oil prices. Our result indicates that both headline and core inflation rise marginally, while oil inflation falls by a significant percentage point around period 2. The oil shock impacts inflation via income and exchange rate channels. The income effect sets in as oil output (export) shrinks and aggregate income falls. Inflationary pressure mounts as aggregate output declines.

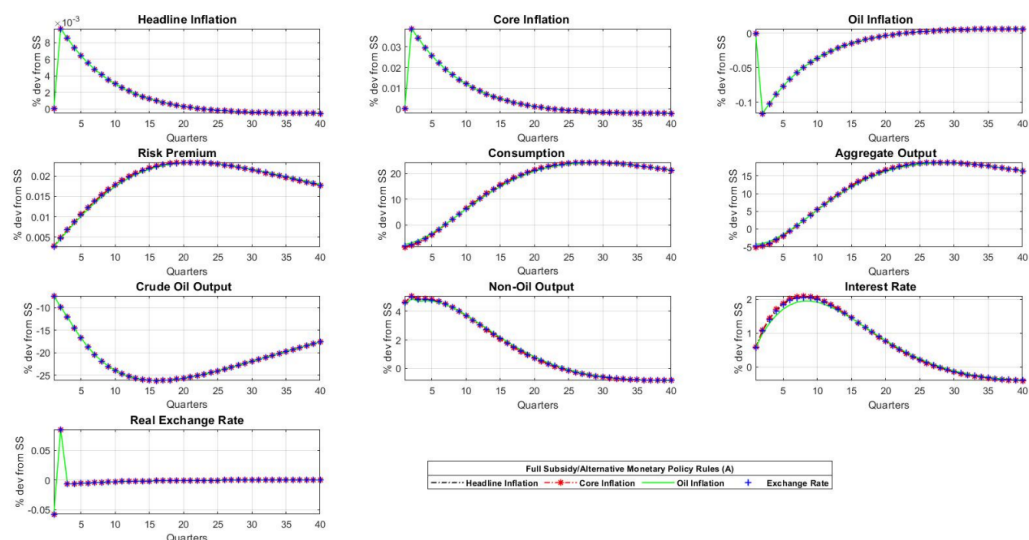


Figure 2a: Negative oil price shock under full subsidy regime

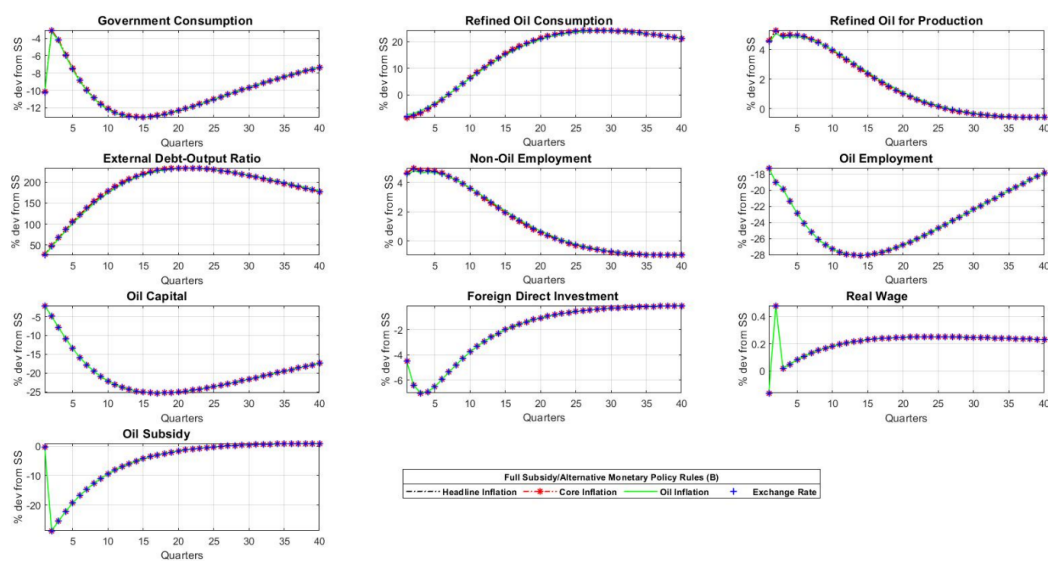


Figure 2b: Negative oil price shock under full subsidy regime

This reflects the inflation component of the oil price-induced stagflation, given that aggregate output declined on impact while headline and core inflation rise after a period lag. In addition, the significant negative effect of the shock on government revenue¹⁸ leads to a fall in government consumption. The exchange rate passthrough kicks in instantly as both headline and core inflation increased sharply as the negative oil price shock materialised. Consequently, aggregate consumption declines, affecting both imported and domestic goods consumed by government and households, as inflationary pressures mount in the economy.

The shock exposes households to additional tax burden as government augments declining oil tax revenue by raising lump-sum tax and borrowing more domestically. Higher lump-sum tax reduces households' disposable income and thus, constrains their capacity to maintain pre-shock consumption level. At this point, households willing to smooth consumption through foreign debt will experience higher borrowing costs as the economy's risk premium goes up. The higher risk premium is caused by higher external debt to GDP ratio and investor concerns about debt sustainability, in addition to the other prevailing macroeconomic risks. Increased risk premium acts like an aggregate demand shock, thus amplifying the effects of the economic slump.

The negative crude oil price shock precipitates a marked contraction in oil output. The crude oil output contraction can be linked directly to the movement of productive resources away from the oil sector given that both marginal products of labour and imported capital in the sector have fallen because of the shock. The development causes movement of oil workers from oil sector to non-oil sector in search of new employment opportunities, job security and better pay, as wage volatility ensues in the economy. Oil capital exhibits high sensitivity to oil price movement. The shock resulted to decline in foreign direct investments and capital accumulation in the oil sector.

Notwithstanding, the non-oil sector appears to benefit from the labour market slack, resulting from the oil sector slump. The non-oil sector's output expands as it attracts more labour resources from the troubled oil sector. Furthermore, non-oil output re-

¹⁸Ninety percent comes from tax on oil firm's net revenue while only ten percent is attributable to lumpsum tax on households.

ceives additional impetus from higher demand for core exports resulting from higher export competitiveness occasioned by the real exchange rate depreciation. The firm does not benefit directly from the oil price decline as its refined oil cost remain unchanged, since refined oil price changes are absorbed¹⁹ wholly by the government. However, the non-oil firm expands its demand for refined oil input utilizing savings from the loose²⁰ labour market. The increased demand for more imported refined oil by domestic firms may exacerbate foreign exchange and inflationary pressures in the economy.

Under the full subsidy fiscal regime, the negative oil price shock resulted to drop in oil subsidy expenditure by the government after a period lag. This effect is shown by the temporary reduction in the size of the fall in government consumption from ten percent on impact to steady state in period two; before plunging further to 14 percent in period fourteen and remained persistent thereafter. This raises a fundamental question on the welfare benefits of oil subsidy on the economy, considering that household oil consumption, household general consumption and government consumption are all negatively impacted by the shock. The reduction in oil subsidy payments does not seem to have any remarkable impact on agents, as both household and government consumption still fell in the aftermath of the oil price shock. The real wage fell on impact before rising in the second period, owing to the increased labour demand from the non-oil sector and exchange rate depreciation.

By adjusting interest rate upward, the central bank responds mainly to exchange rate and inflationary pressures. This is not unexpected in a situation where the economy is being simultaneously buffeted by decline in oil and aggregate output and a rise in key inflation measures, resulting from an adverse oil price shock. In such situation, the central bank would be expected to accord priority to price stability, being the primary object of monetary policy. As seen in both Figures 2a and 2b, the choice of one monetary policy rule²¹ over another does not seem to matter in a fiscal space

¹⁹Government receives the differentials (revenue) when oil price falls and pays the differential (cost) when oil price rises.

²⁰Workers are less expensive to hire.

²¹These refer to headline inflation targeting, core inflation targeting, oil inflation targeting and exchange rate targeting

that inhibits foreign oil price pass-through to the domestic economy. This is because macroeconomic responses to the oil price shock follow similar patterns under all the alternative monetary policy specifications. Our finding underscores how undermining the full price subsidy regime can be for monetary policy transmission mechanism. The absence of foreign oil price pass-through neutralizes almost totally the endogenous differences between alternative monetary policy strategies in our model. Given that refined oil feature in both household consumption basket and domestic firm’s production function, it is no surprise that the full subsidy-induced distortion to domestic oil price weighs heavily on monetary policy.

4.2 Oil Price shock under partial subsidy regime

Under a partial subsidy regime, the government allows only half of the foreign oil price changes to impact the domestic price of imported refined oil. As shown in Figures 3a and 3b, the positive response of headline and core inflation to the shock happened on impact and gets bigger in period 2 before decelerating to steady state levels in period 6.

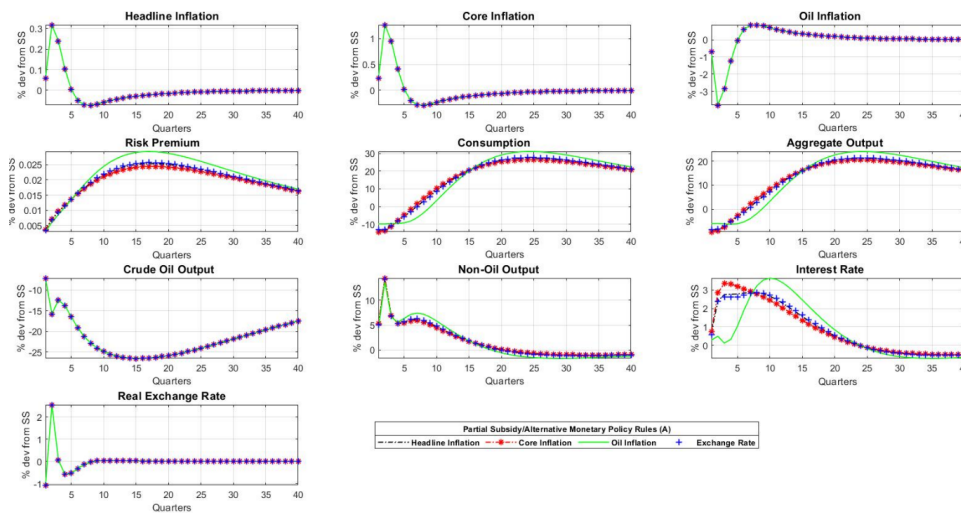


Figure 3a: Negative oil price shock under partial subsidy regime

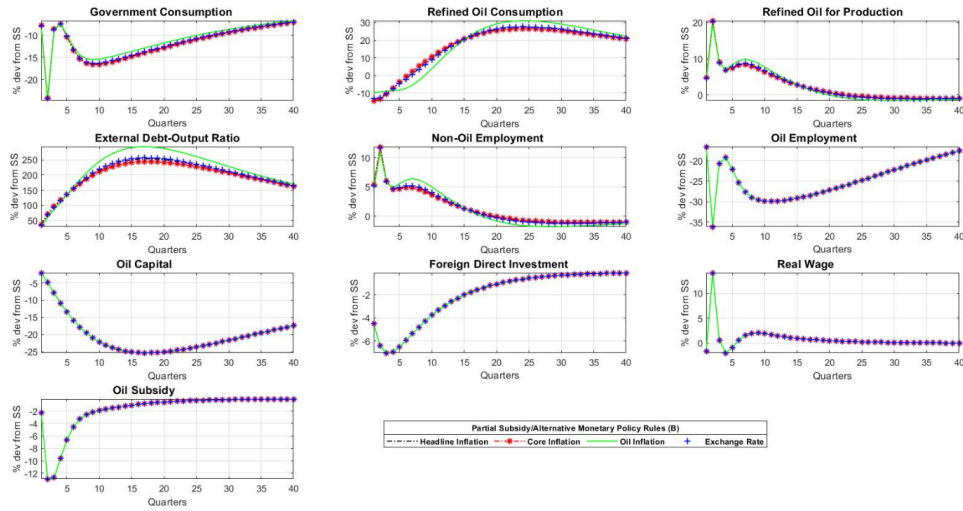


Figure 3b: Negative oil price shock under partial subsidy regime

The responses are faster and larger under the partial subsidy regime compared to the full subsidy regime, owing to the movement from the zero pass-through under full subsidy to a partial pass-through. This shows that the degree of oil price pass-through has effect on inflation’s sensitivity to an oil price shock. However, it did not matter which monetary policy framework was operational as both headline and core inflation measures exhibit similar patterns under the alternative monetary policy rules. Notably, oil inflation response was negative on impact and dipped much further by period 2 before turning positive after period 5. Whereas oil inflation response was mild and negative before returning to steady state under the full subsidy experiment; it showed stronger negative response on impact and in period 2 before turning positive after period 5 under the partial subsidy regime.

The shock elicited instant decline in all consumption variables, arising from the income effect of oil and aggregate output downturn. The variables received stronger impact from a negative oil price shock under the partial subsidy regime, compared to the full subsidy regime. Under full subsidy regime, oil price shocks passthrough is muted due to the fiscal intervention. The monetary policy rule with oil inflation target is associated with less sharp oil consumption, aggregate consumption, and aggregate output response on impact. The headline, core and exchange rate targeting monetary

policy rules are linked to stronger oil consumption, aggregate consumption, and aggregate output negative response to the oil shock. Crude oil output declines under a partial subsidy regime regardless of the monetary policy rule in place.

In response to the shock, non-oil output rose due to the combined effects of the non-oil labour employment increase and the second period real exchange rate depreciation. The two factors improved the non-oil firm's domestic and foreign competitiveness, respectively. It is observed also that the spike in its demand for more workers in period 2 may not be unconnected with the rise in foreign demand, buoyed by the exchange rate depreciation. Furthermore, the non-oil firm is shown to demand more refined oil input for production under a partial subsidy regime because it enjoys at least half of the benefit in the fall in oil factor cost. Similarly, its labour cost declines given that the oil sector sheds more workers under the partial subsidy regime than under the full subsidy regime. The fall in the real wage, on impact, is far more pronounced under the partial subsidy regime than the full subsidy regime; and so was the period two increase. With less subsidy intervention, the firm's adjustment to labour market dynamics, foreign demand and exchange rate developments appears sharper, as reflected by its demand for more workers, more refined oil and expansion in its output.

Given the shock, crude oil output declines, reflecting the loss of oil sector workers and the reversal of oil capital accumulation, owing to a massive decline in foreign direct investments into the oil sector. Whereas oil capital and foreign direct investment are shown to be indifferent to the degree of pass-through as they maintain similar pattern of responses as under the zero pass-through scenario, oil output and oil employment exhibits stronger negative and volatile responses under the partial pass-through policy. For most variables, the shock elicits similar macroeconomic responses under the different monetary policy rules except the oil inflation targeting rule under which many variables exhibit increased volatility. Tracking oil inflation is bound to be problematic given that the underlining variable (i.e. oil price) is volatile and exogenous.

The response of real exchange rate is higher and more persistent under a partial subsidy regime, as it appreciates by 1 percentage point on impact before depreciating

by about 2.5 percent in period 2. Exchange rate volatility persisted till period 8; unlike under the full subsidy regime where exchange rate volatility was contained in size and duration. Households external borrowing condition tightened as risk premium rose in response to the increasing external debt to GDP ratio and dwindling oil revenue. The most profound difference in the impulse responses under the different monetary policy rules is from interest rate response to the shock. Monetary policy response is most aggressive under the core inflation targeting rule and followed by its response under the headline and exchange rate targeting rules. Interest rate response is least aggressive on impact under oil inflation targeting rule, but it turns aggressive over time. The initial slow response to the shock under an oil inflation targeting rule may suggest a temporary modest success in anchoring inflation expectations. The result imply that the central bank will switch to a more aggressive monetary policy under the oil inflation rule whenever oil inflation threat become elevated. Such a policy environment will be a very busy one given the volatile nature of oil price.

4.2 Oil Price shock under zero subsidy regime

The zero-subsidy regime simulation results presented in Figures 4a and 4b show that the transmission mechanism of an exogenous oil price shock is more visible in the domestic economy in the absence of subsidy distortion.

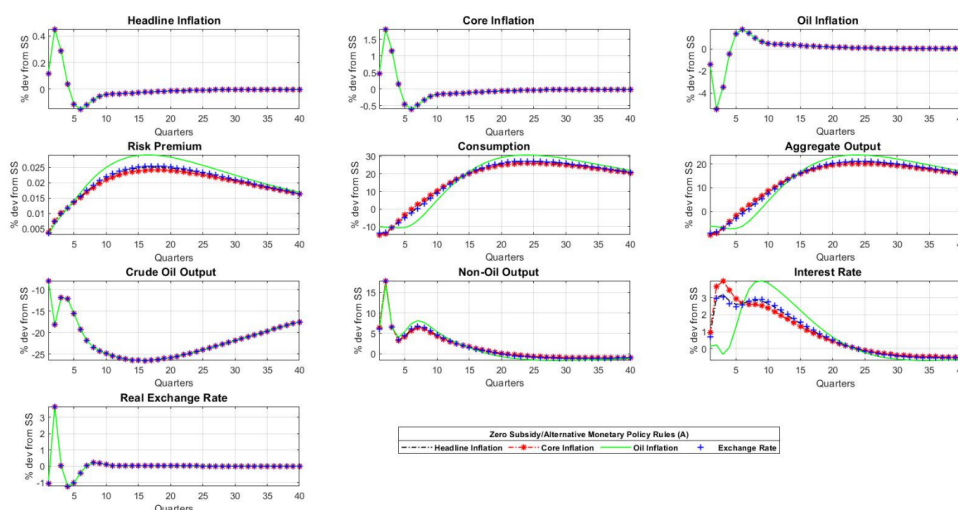


Figure 4a: Negative oil price shock under zero subsidy regime

Accordingly, inflation’s response to the shock is strongest in the absence of oil subsidy. For headline and core inflation, the impact of the negative oil price shock was immediate, positive, and significant up to period 2 before decelerating sharply and returning to steady state in period 15. Similarly, the negative oil inflation response was most significant on impact compared with responses under other subsidy regimes. It moved from about 2 percent fall on impact to about 6 percent in period 2, before recording a minimal positive change in period 5 and finally returned to steady state in period 8. However, there are not clear differences in the responses of inflation and exchange rate to the shock under the alternative monetary policy rules. The situation renders the type of inflation measure or variables targeted in the monetary policy reaction function largely inconsequential for price movement and the real exchange rate. Perhaps, an optimal policy exercise based on optimized simple rules will make the underlining differences between the alternative monetary policy rules less obscure.

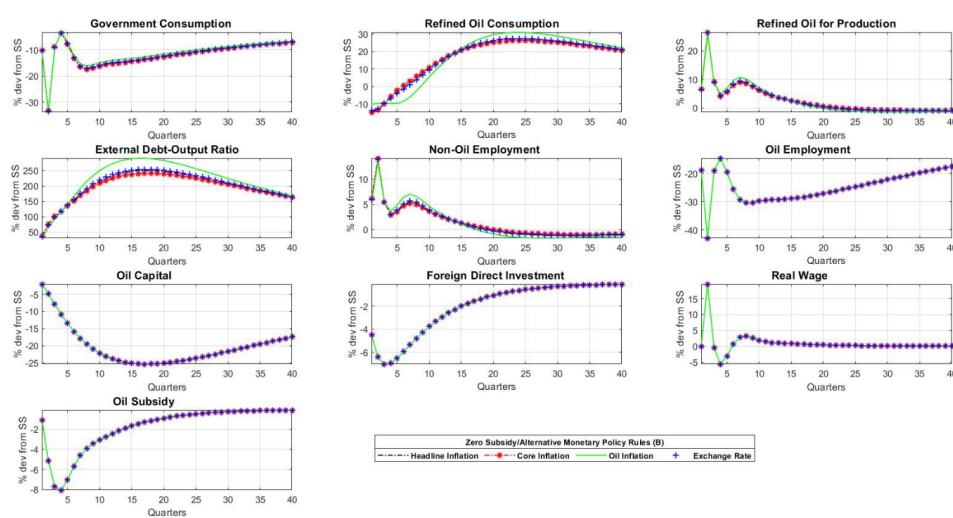


Figure 4b: Negative oil price shock under zero subsidy regime

Aggregate consumption, oil consumption and aggregate output declined in much the same fashion as under partial subsidy regime; with the response under oil inflation policy rule trailing the rest in terms of response size; and beating the rest in terms of volatility. Government consumption fall was more remarkable under the zero-

subsidy regime. It fell by 10 percent on impact, dipped further by an additional 25 percent in period 2 and the effect lingered for much longer. The huge decline, though a primary consequence oil tax revenue loss, is compounded by the lack of fiscal opportunity to accrue savings from negative oil subsidy because there is no subsidy under the fiscal regime.

Non-oil firms produce more under the zero-subsidy regime than under the full and partial subsidy regimes, although its output is more volatile especially under the oil inflation rule. Core output growth reflects the effects of increased labour employment and higher refined oil utilization in the sector, as well as the increased competitiveness brought about by exchange rate depreciation. The firm benefits from fall in the unregulated price of refined oil, workers lay-offs or movement from the depressed oil sector and the greater demand from abroad. Conversely, output fell on impact by the same margin as under previous fiscal regimes, but it comes with higher volatility under the oil inflation rule and the partial and the zero subsidy regimes. The oil output dip is a direct consequence of negative oil price shock which decreased oil sector's productivity leading to fall in oil sector employment, economy-wide wage volatility, fall in foreign direct investment and oil capital decumulation.

The real exchange rate is most volatile under the zero-subsidy market condition, while both risk premium and external debt-to-GDP ratio increased as well. The tightening of external borrowing condition reflects macroeconomic vulnerability due to negative oil price shock, decline in oil earnings, output slump and constrained consumption, among others. The interest rate response to inflationary pressures caused by oil shock is most aggressive under core inflation targeting and least aggressive but most volatile under the oil inflation targeting rule. Given a complete foreign oil price pass-through to the domestic economy, the oil inflation targeting rule can not elicit an aggressive interest rate response on impact as the economy experiences decrease in oil inflation. However, the income effect of the negative oil price shock, manifesting through decline in foreign exchange earnings and exchange rate depreciation, core inflation rises significantly, prompting an aggressive interest rate response under the core inflation targeting policy rule.

In summary, our results show that the magnitude of macroeconomic responses to the

negative oil price shock are comparably smaller under a zero pass-through fiscal policy scenario²² than under both the partial and full pass-through policies. This result corresponds to Bouakez *et al.* (2008) who reported that under a full subsidy policy, the transmission of a positive oil price shock to the domestic economy is subdued, since subsidy act as a smoother of macroeconomic responses to the shock.

5. Optimal Monetary Policy

We implement the optimized simple rules for the four (4) alternative monetary policy rules given the model's equilibrium conditions and a negative crude oil price shock. The experiment is subject to three (3) possible oil subsidy regimes; viz: (i) full subsidy; (ii) partial subsidy; and (iii) zero subsidy. Under each of the fiscal regimes, the coefficients of each of the Taylor-type policy rules are optimized²³ to produce minimum variances associated with an ad-hoc expected loss function. The expression for the variance of such the loss function is as follows:

$$E(L_t) = \lambda_\pi Var(\pi_t) + \lambda_y Var(y_t) + \lambda_r Var(i_t) + \lambda_s Var(q_t) \quad (55)$$

This represents the weighted average of the unconditional variances of inflation, output, exchange, and interest rates. The weights (λ_π , λ_y , λ_q and λ_i) are chosen optimally to achieve quadratic loss function minimization. Policy preferences or weights combination(s) that delivers the minimum loss value post optimization under the alternative monetary policy specifications is deemed to be most welfare superior. In addition, the target instrument under which the minimum loss value is derived will be considered most appropriate for adoption given the state of things in the economy. We follow Alpanda *et al.* (2010), Hove *et al.* (2015) and Ferrero & Seneca (2015) to formulate the relative loss function weights that reveals the policy maker's preferences of 0.5 to 2; resulting in ten (10) alternative plausible weight combinations. Inflation variance weight is normalized to one (1) as it is done in the literature. Each weighted loss function is minimized under each of the alternative simple policy rules to compute the values of central bank losses. Results of the policy exercise are pre-

²²The full subsidy regime

²³Using Dynare's toolbox for optimized simple rules (OSRs) ²⁹This refers to the central bank's preference No. 1.

sented in tables 3 - 6 in the sections that follow.

5.1 Policy loss under full oil subsidy

Allowing for full subsidy or a zero oil price pass-through, optimized simple rules (OSR) welfare exercise conducted following a negative oil price shock suggests as shown in Table 3 that, either core inflation targeting (CIT) or oil inflation targeting (OIT) will pass for the optimal policy under the policy maker's preference that normalizes inflation to unity and attaches a uniform weight of 0.5 to other variables²⁶ and the one that maintains the same weight allocations for other variables while assigning 1 to the exchange rate²⁴. In terms of the appropriate weighting for each variable in the loss function, optimal policy favours low weights on output and interest rate, while exchange rate weight can be slightly higher and same with inflation weight. However, any exchange rate weight above 1 comes at a significant welfare cost.

Table 3: Loss values under a full subsidy regime

S/N	Central Bank's Preferences				Loss values under alternative rules			
	λ_π	λ_y	λ_i	λ_q	HIT	CIT	OIT	ERT
1	1	0.5	0.5	0.5	1.2135	1.2134	1.2134	1.2843
2	1	1	0.5	0.5	2.291	2.2612	2.2612	2.4652
3	1	1.5	0.5	0.5	3.2973	3.4097	3.2971	3.6349
4	1	2	0.5	0.5	4.3358	4.3283	4.3283	4.8002
5	1	0.5	1	0.5	1.3421	1.3421	1.3421	1.3615
6	1	0.5	1.5	0.5	1.4102	1.4102	1.4102	1.4212
7	1	0.5	2	0.5	1.4632	1.4631	1.4631	1.4712
8	1	0.5	0.5	1	1.2138	1.2134	1.2134	1.2844
9	1	0.5	0.5	1.5	1.2149	1.2135	1.2135	1.2844
10	1	0.5	0.5	2	1.2136	1.2135	1.2135	1.2845

The policy maker's preference that delivers the biggest welfare losses across all monetary policy specifications is the one that assigns maximum weight to output. Any output weight value above the minimum of 0.5 will have adverse welfare consequences. This imply that, given a negative oil price shock realization in a full subsidy environment, it is not optimal for the central bank to be aggressive about minimiz-

²⁴This refers to the central bank's preference No. 8

ing output volatility. Under the circumstance, the worst possible policy choice is to target the real exchange rate in the Taylor rule while trying to minimize output variance in the central bank’s loss function in an aggressive fashion²⁵. The outcome will be most unfavourable for welfare. Under the zero pass-through scenario, the policy maker may be indifferent in choosing the optimal inflation anchor between core and oil inflation since both yield the same loss value.

5.2 Policy loss under partial oil subsidy

Assuming a partial subsidy regime in the wake of a negative oil price shock, central bank’s preference 1 which assigns the weight of 1 to inflation and 0.5 to other variables is associated with the lowest welfare loss value under oil inflation targeting (OIT) rule. The results, as shown in Table 4 indicates that headline inflation targeting (HIT) trails OIT in terms of welfare performance. Both values are obtained under the same preference combination 1.

Table 4: Loss values under a full subsidy regime

S/N	Central Bank’s Preferences				Loss values under alternative rules			
	λ_π	λ_y	λ_i	λ_q	HIT	CIT	OIT	ERT
1	1	0.5	0.5	0.5	1.8711	2.1547	1.861	2.4465
2	1	1	0.5	0.5	3.537	3.4865	3.4788	4.7545
3	1	1.5	0.5	0.5	5.1678	5.1021	5.0959	7.0548
4	1	2	0.5	0.5	6.7957	6.7158	8.8841	9.3524
5	1	0.5	1	0.5	2.0574	2.5274	2.5066	2.5209
6	1	0.5	1.5	0.5	2.2377	2.5274	2.2344	2.5808
7	1	0.5	2	0.5	2.6694	2.6694	2.6694	2.6321
8	1	0.5	0.5	1	1.9323	1.9057	1.9031	2.4974
9	1	0.5	0.5	1.5	1.9478	1.9465	1.9452	2.5307
10	1	0.5	0.5	2	2.0019	1.994	1.9879	2.5728

The preferences with greater emphasis on output stabilization also turn out to be the worst performing in terms of welfare, under all the Taylor rule specifications. Under the partial pass-through scenario, caring too much about output volatility²⁶ under an exchange rate targeting framework will be most welfare unfriendly. The partial subsidy fiscal policy scenario may be more aligned to reality in most oil rich emerging

²⁵Assigning weight of 2 to output variance in the loss function

²⁶By attaching a high weight (2) to output in the loss function

and frontier small open economies. The common approach is a staggered implementation of subsidy removal, as total oil subsidy removal is often associated with charged social and political response in these economies.

5.3 Policy loss under zero oil subsidy

Results obtained in a world with complete oil price pass-through as in Table 5 show that monetary policy response to a negative oil price shock is most welfare attractive if the policy maker chooses preference 1, which normalizes the weight on inflation to 1 and assigns 0.5 to the other variables in the loss function and uses oil inflation as the policy anchor²⁷.

Table 5: Loss values under a full subsidy regime

S/N	Central Bank's Preferences				Loss values under alternative rules			
	λ_π	λ_y	λ_i	λ_q	HIT	CIT	OIT	ERT
1	1	0.5	0.5	0.5	2.3164	2.2954	2.2927	2.8532
2	1	1	0.5	0.5	5.6185	5.5772	5.6387	5.5547
3	1	1.5	0.5	0.5	6.3844	6.2994	6.2788	8.1971
4	1	2	0.5	0.5	11.0186	8.2868	10.8438	10.8397
5	1	0.5	1	0.5	2.4994	2.4973	2.4947	2.9259
6	1	0.5	1.5	0.5	3.0629	3.0629	3.0629	2.9828
7	1	0.5	2	0.5	3.114	3.114	3.114	3.0353
8	1	0.5	0.5	1	2.418	2.384	2.3792	2.9406
9	1	0.5	0.5	1.5	2.4855	3.0595	2.4667	3.028
10	1	0.5	0.5	2	3.1808	2.5597	2.5831	3.1154

The second-best option results from the same policy maker's preference under a core inflation targeting Taylor rule. The worst welfare outcome is attributed to the central bank's preference that places strong weight on output stabilization in the loss function while targeting the real exchange rate in the Taylor rule. Preference 4 in the central bank's loss function weights combinations yield the worst possible welfare results across all Taylor rule specifications and subsidy regimes. This suggest that a net oil exporting small open economy may not achieve optimal policy if it focuses heavily on smoothing output volatility. Output stabilization objective can be more feasible if pursued indirectly through inflation stabilization.

²⁷See table 5

6. Conclusion

With a medium size New Keynesian small open economy DSGE model, we capture salient structural features of net oil exporting developing and emerging economies. The model highlights conditions which make less-developed net oil exporters more susceptible to adverse external events. Using standard small open economy DSGE model parameter values and steady state ratios obtained based on Nigerian data, we demonstrate, through simulation, how a negative oil price shock impacts business cycle variables under alternative monetary and fiscal policy arrangements.

A negative oil price shock sets off a chain of macroeconomic reactions that saw oil output, aggregate income, government revenue and consumption fall while a reverse Dutch disease situation led to improvement in the non-oil sector output performance. Given a full oil subsidy fiscal regime, monetary policy is shown to be indifferent to alternative target variables as macroeconomic responses to a negative oil price shock are similar across the alternative monetary policy rules. As subsidy intervention tends toward zero, and pass-through approaches unity, macroeconomic volatility in response to the shock increases. Oil inflation targeting is associated with the least aggressive monetary policy reaction to the shock but exhibits higher volatility at longer term horizons. Optimal monetary policy exercise reveals that oil inflation targeting has the most welfare gain. These results may have been influenced by our model characterization which captures important stylized facts of a net oil exporter in which refined oil feature as both a consumption good and a production factor. Notably, the argument by Natal (2012) that oil price changes operate as a distortionary tax on disposable income and a source of monetary policy trade-off amplification seems to hold in our model. Given the income effect of a negative oil price shock and the impact of low elasticity of substitution between oil and core goods, stabilizing oil inflation is revealed to be welfare-enhancing.

A monetary policy strategy that targets oil inflation is bound to be fraught with daunting policy challenges. Oil price, the underlining variable for oil inflation, is exogenous and highly volatile and to conduct monetary policy based on its evolution may undermine monetary policy inertia, give rise to dynamic inconsistency problem, and erode central bank's credibility, an asset it requires for monetary policy success. Per-

haps, this explains why existing models²⁸ that feature oil generally abstract from anchoring oil inflation directly when setting up alternative Taylor rules to be optimized. The common practice in the literature is to consider core inflation as the approximate inflation measure to respond to in the event of a persistent oil shock. In our model, the core inflation anchor delivers results nearly equivalent to oil inflation anchor, and since the costs of targeting a largely exogenous oil inflation directly cannot be reasonably conceived, the recommended optimal path for practical policy purposes will be to target core inflation. Our proposition aligns with Aissa & Rebei (2012) who recommends that core inflation should be targeted by central banks of economies with regulated prices given that it excludes distortions arising from administered prices.

Monetary policy alone cannot address supply side problems and the structural deficiencies which predispose net oil exporters to external shocks. To that end, net oil exporting emerging and developing countries must improve domestic productivity through industrialization, ensure proper forward and backward linkages between domestic oil sector and the rest of their economies to maximize the benefits of oil endowment, commit to fiscal rules that de-link government fiscal operations from direct oil revenue performance, ensure strong monetary-fiscal policy coordination and re-calibrate their economies to achieve diversification and self-sufficiency in critical sectors.

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²⁸See Medina & Soto (2005), Bouakez et al. (2008) and Allegret & Benkhodja (2015)

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Appendix

Log-linearized equations

Household:

$$\begin{aligned}\widetilde{c}_t^h &= (\varpi\Psi_f)\widetilde{Y}_t + (-\nu\Psi_{ro})(\widetilde{p}_t^c - \widetilde{p}_t^{ro}) + \widetilde{c}_t \\ \widetilde{c}_t^f &= -\$(1 - \Psi_f)\widetilde{Y}_t + (-\nu\Psi_{ro})(\widetilde{p}_t^c - \widetilde{p}_t^{ro}) + \widetilde{c}_t \\ \widetilde{c}_t^{ro} &= -\nu(1 - \Psi_{ro})(\widetilde{p}_t^{ro} - \widetilde{p}_t^c) + \widetilde{c}_t \\ \widetilde{c}_t &= E_t\widetilde{c}_{t+1} - \frac{1}{\eta}(\widetilde{i}_t - E_t\pi_{t+1})\end{aligned}$$

Domestic Goods Firms

$$\begin{aligned}\widetilde{y}_t^h &= \widetilde{z}_t^h + \#_h\widetilde{ro}_t^h + (1 - \#_h)\widetilde{l}_t^h \\ \widetilde{ro}_t^h - \widetilde{l}_t^h &= \omega_h(\widetilde{w}_t - \widetilde{p}_t^{ro}) \\ \widetilde{nmc}_t^h &= -\frac{1}{\omega_h}\widetilde{z}_t^h + \#_h(\widetilde{p}_t^{ro} - \widetilde{p}_t^h) + (1 - \#_h)\widetilde{w}_t \\ \widetilde{rmc}_t^h &= -\frac{1}{\omega_h}\widetilde{z}_t^h + \#_h(\widetilde{p}_t^{ro} - \widetilde{p}_t^h) + (1 - \#_h)\widetilde{w}_t - (\widetilde{p}_t^h - \widetilde{p}_t) \\ \widetilde{z}_t^h &= \rho_{z^h}\widetilde{z}_{t-1}^h + \varepsilon_t^{z^h} \dots \dots \dots \varepsilon_t^{z^h} \sim N(0, \sigma_{z^h}^2) \\ \widetilde{\pi}_t^h &= (1 - \theta^h)\left(\overset{reset}{\widetilde{p}_t^h} - \widetilde{p}_{t-1}^h\right) + (\theta^h)^2\widetilde{\pi}_{t-1}^h \\ \overset{reset}{\widetilde{p}_t^h} - \widetilde{p}_{t-1}^h &= \beta\theta^h E_t(\widetilde{\pi}_{t+1}^h) + \widetilde{\pi}_t^h + (1 - \beta\theta^h)\widetilde{rmc}_t^h \\ \widetilde{\pi}_t^h &= (1 - \beta\theta^h)E_t(\widetilde{\pi}_{t+1}^h) + \theta^h\widetilde{\pi}_{t-1}^h + \kappa_t^h\widetilde{rmc}_t^h\end{aligned}$$

Imports Price Setting and Incomplete pass-through

$$\widetilde{\pi}_t^f = (1 - \beta\theta^f)E_t(\widetilde{\pi}_{t+1}^f) + \theta^f\widetilde{\pi}_{t-1}^f + \kappa_t^f\widetilde{\Psi}_t$$

Inflation Aggregation

$$0 = r\widetilde{p}_t^{ro} + (1 - \Psi_{ro})(1 - \Psi_f)\widetilde{r}p_t^h + (1 - \Psi_{ro})\Psi_f r\widetilde{p}_t^f$$

Exchange Rate, Terms of Trade and Foreign Demand

$$\widetilde{q}_t = \widetilde{s}_t + \widetilde{p}_t^{f*} - \widetilde{p}_t$$

$$\widetilde{s}_t = \widetilde{s}_{t-1} + \widetilde{\pi}_t^f - \widetilde{\pi}_t^* + \widetilde{\Psi}_t - \widetilde{\Psi}_{t-1}$$

$$\widetilde{Y}_t = \widetilde{p}_t^f - \widetilde{p}_t^h$$

$$\widetilde{c}_t^{h*} = -\epsilon_h^* (\widetilde{p}_t^h - \widetilde{p}_t - \widetilde{q}_t) + \widetilde{c}_t^*$$

Imperfect International Risk Sharing and Uncovered Interest Rate Parity

$$\widetilde{c}_t = \frac{1}{\eta} (\widetilde{q}_t + \widetilde{F}_t) + \widetilde{c}_t^*$$

$$\widetilde{r}_t - \widetilde{r}_t^* = E_t(\widetilde{q}_{t+1} - \widetilde{q}_t) + \widetilde{F}_t$$

External Debt and Risk Premium

$$\widetilde{F}_t = \Phi_d \widetilde{d}_t + \epsilon_t^F$$

Domestic Oil Sector and Foreign Demand for Oil

$$\widetilde{p}_t^{co*} = \rho_{p^{co*}} \widetilde{p}_{t-1}^{co*} + \epsilon_t^{p^{co*}}$$

$$\widetilde{fdi}_t = \rho_{fdi} \widetilde{fdi}_{t-1} + (1 - \rho_{fdi}) \widetilde{p}_t^{co*} + \epsilon_t^{fdi}$$

$$\widetilde{y}_t^{co} = \widetilde{z}_t^{co} + \gamma_{co} \widetilde{k}_t^{co} + (1 - \gamma_{co}) \widetilde{l}_t^{co}$$

$$\widetilde{l}_t^{co} = \widetilde{p}_t^{co} + \widetilde{y}_t^{co} - \widetilde{w}_t \widetilde{r}_t^{co} = \widetilde{p}_t^{co} + \widetilde{y}_t^{co} - \widetilde{k}_t^{co} \widetilde{k}_{t+1}^{co} = \frac{\overline{FDI}}{\overline{K}^{co}} (\widetilde{fdi}_t) + (1 - \delta_{co}) \widetilde{k}_t^{co} \widetilde{nr}_t^{co}$$

$$= \left(1 / \frac{\overline{NR}^{co}}{\overline{GR}^{co}} \right) \widetilde{gr}_t^{co} - \left(\frac{\overline{WL}^{co}}{\overline{GR}^{co}} / \frac{\overline{NR}^{co}}{\overline{GR}^{co}} \right) (\widetilde{w}_t + \widetilde{l}_t^{co}) - \left(\frac{\overline{R}^{co} \overline{K}^{co}}{\overline{GR}^{co}} / \frac{\overline{NR}^{co}}{\overline{GR}^{co}} \right) (\widetilde{r}_t^{co} + \widetilde{k}_t^{co}) \widetilde{z}_t^{co} = \rho_{z^{co}} \widetilde{z}_{t-1}^{co} + \epsilon_t^{z^{co}}$$

$$\widetilde{c}_t^{hco*} = -\epsilon_{co}^* (\widetilde{p}_t^{co} - \widetilde{p}_t - \widetilde{q}_t) + \widetilde{c}_t^{co*}$$

Labour Market

$$\widetilde{w}_t = \widetilde{nm}c_t^h + \widetilde{l}_t^{co} - \widetilde{p}_t^{co} - \widetilde{y}_t^{co} + \frac{1}{\omega_h} \left[(1 - \#_h) \widetilde{y}_t^h - \widetilde{l}_t^h \right]$$

Government Behaviour and Fiscal Rule

$$\widetilde{g}_t^h = v_g \Psi_g \widetilde{Y}_t + \widetilde{g}_t$$

$$\widetilde{g}_t^f = -v_g (1 - \Psi_g) \widetilde{Y}_t + \widetilde{g}_t$$

$$P_t^c = \left[(1 - \Psi_g) (P_t^h)^{1-v_g} + \Psi_g (P_t^f)^{1-v_g} \right]^{\frac{1}{1-v_g}}$$

$$\widetilde{g}_t = \rho_g \widetilde{g}_{t-1} + \epsilon_t^g \dots \dots \epsilon_t^g \sim N(0, \sigma_g^2)$$

$$\widetilde{\tau}_t = \rho_\tau \widetilde{\tau}_{t-1} + (1 - \rho_\tau) (\chi_1 \widetilde{d} - \chi_2 (\widetilde{p}_t^{co} + \widetilde{y}_t^{co} - \widetilde{p}_t)) + \epsilon_t^\tau$$

Refined Oil Price and Subsidy

$$P_t^{ro} = \varsigma P_{t-1}^{ro} + (1 - \varsigma) S_t P_t^{ro*}$$

$$\widetilde{\theta}_t = \frac{\overline{QP}^{ro*}}{\Theta} (\widetilde{q}_t + \widetilde{p}_t^{ro*}) - \frac{\overline{RP}^{ro}}{\Theta} (\widetilde{r} \widetilde{p}^{ro}_t)$$

Crude Oil Price, Refined Oil Price and Foreign Value Addition

$$\widetilde{p}_t^{ro*} = \zeta_{p^{co*}} \widetilde{p}_{t-1}^{co*} + (1 - \zeta_{p^{ro*}}) \widetilde{va}_t^{ro*} + \epsilon_t^{ro*}$$

$$\widetilde{va}_t^{ro*} = \rho_{va^{ro*}} \widetilde{va}_{t-1}^{ro*} + \epsilon_t^{va^{ro*}}$$

Aggregation and Market Clearing

$$Y_t = Y_t^h + X_t - M_t$$

$$Y_t^h = C_t^h + C_t^{h*} + G_t^h$$

$$Y_t^{hco} = C_t^{hco*}$$

$$M_t = M_t^f + M_t^{ro}$$

$$M_t^f = C_t^f + G_t^f$$

$$M_t^{ro} = C_t^{ro} + RO_t^h$$

$$X_t = C_t^{h*} + Y_t^{hco}$$

$$\begin{aligned}
 B_t^{agg} &= B_t^* \\
 L_t &= L_t^h + L_t^{co} \\
 \widetilde{y}_t^h &= \frac{\overline{C^h}}{\overline{Y^h}} \widetilde{c}_t^h + \frac{\overline{C^{h*}}}{\overline{Y^h}} \widetilde{c}_t^{h*} + \frac{\overline{G^h}}{\overline{Y^h}} \widetilde{g}_t^h \\
 \widetilde{y}_t &= \frac{\overline{C}}{\overline{Y}} \widetilde{c}_t + \frac{\overline{X}}{\overline{Y}} \widetilde{x}_t - \frac{\overline{M}}{\overline{Y}} \widetilde{m}_t \\
 \widetilde{x}_t &= \frac{\overline{C^{h*}}}{\overline{X}} \widetilde{c}_t^{h*} + \frac{\overline{C^{hco*}}}{\overline{X}} \widetilde{c}_t^{hco*} \\
 \widetilde{m}_t &= \frac{\overline{M^f}}{\overline{M}} \widetilde{m}_t^f + \frac{\overline{M^{ro}}}{\overline{M}} \widetilde{m}_t^{ro} \\
 \widetilde{m}_t^f &= \frac{\overline{C^f}}{\overline{M^f}} \widetilde{c}_t^f + \frac{\overline{G^f}}{\overline{M^f}} \widetilde{g}_t^f \\
 \widetilde{m}_t^{ro} &= \frac{\overline{C^{ro}}}{\overline{M^{ro}}} \widetilde{c}_t^{ro} + \frac{\overline{RO^h}}{\overline{M^{ro}}} \widetilde{ro}_t^h \\
 \widetilde{l}_t &= \left(\frac{\overline{L^h}}{\overline{L}} \right) \widetilde{l}_t^h + \left(\frac{\overline{L^{co}}}{\overline{L}} \right) \widetilde{l}_t^{co}
 \end{aligned}$$

External Debt and Current Account Dynamics

$$\widetilde{b}_t^* = \frac{1}{\beta} \widetilde{b}_{t-1}^* + \widetilde{nx}_t - f \widetilde{dir}_t + \frac{\overline{NX/PY}}{\beta - 1} \left(\widetilde{r}_{t-1}^* + \widetilde{\Psi}_{t-1} + \Delta \widetilde{q}_t - \widetilde{\pi}_t \right) \tag{56}$$

$$\widetilde{nx}_t = \widetilde{x}_t - \widetilde{m}_t f \widetilde{dir}_t = 1 - \tau^{co} \left(nr_t^{co} \right)$$

Monetary Policy

$$\begin{aligned}
 \widetilde{i}_t &= \rho_i \widetilde{i}_{t-1} + (1 - \rho_i) \left(\$1 \widetilde{y}_t + \$2 \widetilde{\pi}_t + \$3 \widetilde{\pi}_t^h + \$4 \widetilde{\pi}_t^c + \$5 \Delta \widetilde{s}_t \right) + \epsilon_t^i \\
 \widetilde{i}_t &= \rho_i \widetilde{i}_{t-1} + (1 - \rho_i) (\$1 \widetilde{y}_t + \$2 \widetilde{\pi}_t) + \epsilon_t^i \widetilde{i}_t = \rho_i \widetilde{i}_{t-1} + (1 - \rho_i) \left(\$1 \widetilde{y}_t + \$3 \widetilde{\pi}_t^h \right) + \epsilon_t^i \widetilde{i}_t \\
 &= \rho_i \widetilde{i}_{t-1} + (1 - \rho_i) (\$1 \widetilde{y}_t + \$4 \widetilde{\pi}_t^c) + \epsilon_t^i \widetilde{i}_t = \rho_i \widetilde{i}_{t-1} + (1 - \rho_i) (\$1 \widetilde{y}_t + \$5 \Delta \widetilde{s}_t) + \epsilon_t^i
 \end{aligned}$$

Foreign Economy

$$\widetilde{i}_t^* = \rho_i^* \widetilde{i}_{t-1}^* + \epsilon_t^{i*}$$

$$\widetilde{c}_t^* = \rho_{c^*} \widetilde{c}_{t-1}^* + \epsilon_t^{c^*}$$

$$\widetilde{c}_t^{co*} = \rho_{c^{co*}} \widetilde{c}_{t-1}^{co*} + \epsilon_t^{c^{co*}}$$

$$\widetilde{\pi}_t^* = \rho_{\pi^*} \widetilde{\pi}_{t-1}^* + \epsilon_t^{\pi^*}$$

$$\widetilde{p}_t^{f*} = \rho_{p^{f*}} \widetilde{p}_{t-1}^{f*} + \epsilon_t^{p^{f*}} \widetilde{p}_t^{co*} = \rho_{p^{co*}} \widetilde{p}_{t-1}^{co*} + \epsilon_t^{p^{co*}}$$